Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 266 OCTOBER, 1958 No. 4

INERTIAL NAVIGATION

BY

C. F. O'DONNELL [1]

Part I [2]

ABSTRACT

A brief survey of inertial guidance characteristics and their bearing on the suitability of inertial guidance for particular missions is followed by a development of the Schuler tuning of inertial platforms for error limiting. A comparison between inertial guidance requirements for navigation and ballistic trajectories is made. Forms of mechanization are described and their advantages and disadvantages listed. Problems of inertial guidance, such as the choice of coordinate systems, alignment, and stabilization, are outlined. Important characteristics of subsystems such as the inertial components and computers are presented. Methods of error analysis are developed and a sample system error analysis worked. A brief description of the use of auxiliary position and velocity information in aided inertial systems concludes the report.

A. GUIDANCE PROBLEMS SUITED FOR INERTIAL GUIDANCE

Inertial guidance systems readily supply information on vehicle velocity with respect to the Earth, present position, and vehicle attitude. Guidance problems requiring this information present likely areas for the application of inertial guidance. In some cases, such as that of a mobile destination, additional equipment may be required to solve the guidance problem, bringing in one form of an aided inertial system. Hybrid systems of this type, however, have been considered to be outside the scope of this paper.

Individual cases will be concerned with the state-of-the-art in meeting cost, weight, volume, and accuracy requirements. This detailed information is usually classified, but we can examine the characteristics of inertial guidance in more detail than is included in our opening sentence.

[1] Autonetics, Division of North American Aviation, Inc., Downey, Calif.

[2] Part II will appear in this JOURNAL for November 1958.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

First, inertial guidance supplies a self-contained "dead reckoning" type of navigation. This method of navigation has been used for many years (Fig. 1). Given initial position, a ship's log to determine speed, a compass for heading, and a time standard, a ship's position and course to destination can be computed as a continuous function of time. There is a fundamental difference: the marine navigation system described measured velocity with respect to the water's surface, so even for a perfect mechanization a surface drift of 1 knot leads to a 24 nautical mile per day navigation error; the inertial guidance system uses inertial space as its reference, and so is independent of these surface phenomena. The inertial system requires no weather or ocean current data, no detectable radiation, no magnetic compass, no time-shared usage of ground facilities, no operator time during flight, and no special maps. Its accuracy, which is independent of operating altitude and terrain, is limited almost solely by the accuracies of its component instruments.

Second, initial conditions, both position and velocity, must be set into the inertial guidance system. Moreover, following performance

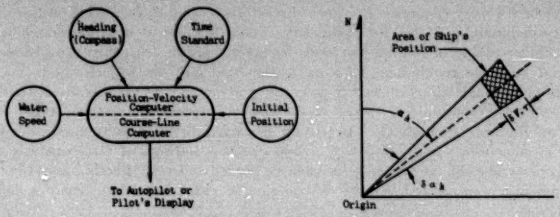


FIG. 1. Dead reckoning navigation.

depends directly on the accuracy with which the initial conditions are established, for in an inertial navigation system initial condition errors propagate as undamped sinusoidal oscillations.

Third, time rather than distance travelled is the major criterion in determining error build-up. In general, the probable position error grows with time and, for intervals much greater than the 84-min. period natural to the system, this growth is approximately proportional to the square root of time.

It would appear that inertial guidance could be considered suitable when high accuracy is required for a comparatively short operating time without regard to range covered, or when medium accuracy is required over long periods of operating time with a square-root-of-time error build-up being allowable.

Even in applying these general characteristics caution must be observed. For example, consider the errors shown in Fig. 2. The $k(1 - \cos \omega t)$ error propagation (Fig. 2*a*)—due, possibly, to a sudden tilt of the platform resulting from an acceleration impulse at the start

of the navigation period—is not growing with time but does reach a peak in the relatively short period of 42 min. In simple models of inertial navigators a constant gyro drift rate produces a uniformly increasing position error (Fig. 2*b*). The result of adding these two errors, using a more compressed time scale, is shown in Fig. 2*c*. Initially the (1 − cos) error could be dominant, so for short-term usage this type of error could determine the system accuracy. After a sufficiently long time the *kt* error is dominant, so that after several 84-min. periods have passed this term could be the main term in the system

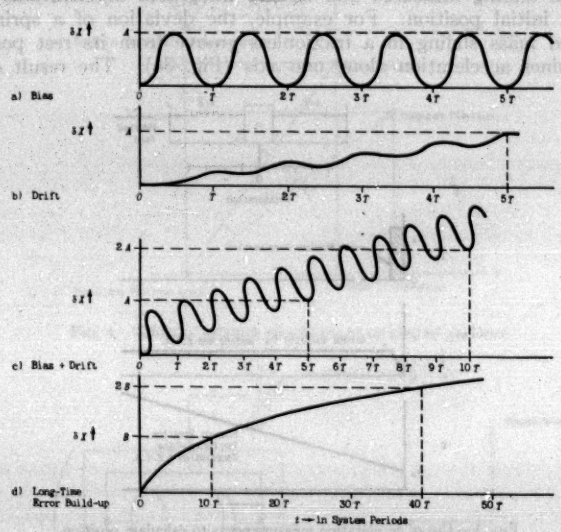


FIG. 2. Error propagation in inertial guidance systems.

position error. The change-over point of dominance between these two error sources will obviously depend on both the system design and usage.

In practice, constant gyro drift is a polite fiction and, when really "long-time" system operation is considered, the drift rate should be considered as a random variable. In that case, the resulting position error builds up as the square root of time (Fig. 2*d*). The definition of "long time" depends on the correlation time for gyro drift among other things.

While it is difficult to make a definite statement about guidance problems suited for inertial guidance in general, this is not of concern to the practicing engineer. In every particular case, knowing the

details of the mission to be accomplished and the characteristics of the components available to him, he can readily determine whether or not inertial guidance is feasible. To show how this can be done is the concern of the remainder of this paper.

B. THE BASIC PRINCIPLES OF INERTIAL GUIDANCE

1. Spatial Reference of Gyroscopes

The intention in an inertial guidance system is to determine position through adding measured and doubly integrated accelerations to a known initial position. For example, the deviation of a spring-restrained mass sliding in a frictionless groove from its rest position determines acceleration along one axis (Fig. 3*a*). The result of an

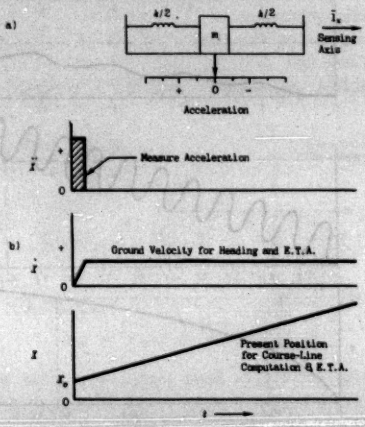


FIG. 3. Use of acceleration measurements to calculate position.

acceleration impulse occurring at $t=0$ along this axis is shown in Fig. 3*b*. The first integration of the impulse gives the step change in velocity while the second integration gives the ramp change in position. The addition of this change to the original value of the coordinate determines present position as a function of time.

Navigation is normally at least a two-dimensional problem. In order to determine position, acceleration must be measured as a vector. For example, consider that navigation is taking place on a flat plane with Cartesian position coordinates X and Y. Then if there is an error ϕ_e in the azimuth orientation of the acceleration measuring device (Fig. 4), calculated and actual positions diverge steadily with distance travelled, even if the guidance system is otherwise perfect.

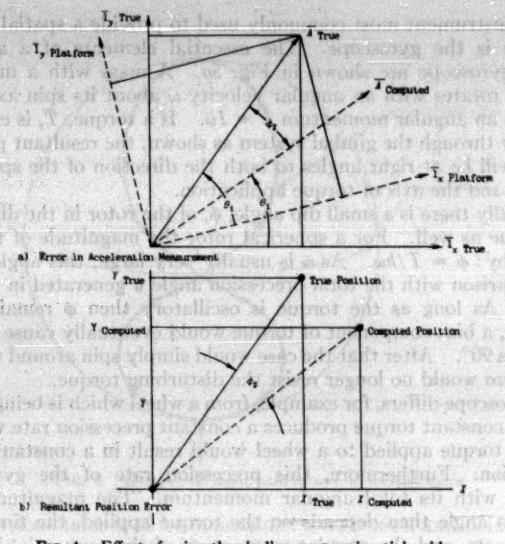


FIG. 4. Effect of azimuth misalignment on inertial guidance.

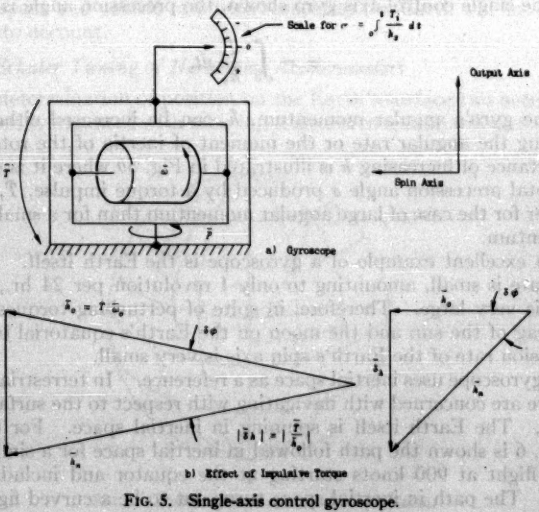


FIG. 5. Single-axis control gyroscope.

The instrument most commonly used to provide a spatial direction reference is the gyroscope. The essential elements of a single-axis control gyroscope are shown in Fig. 5*a*. A mass with a moment of inertia I rotates with an angular velocity ω about its spin axis and so possesses an angular momentum $\bar{h} = I\bar{\omega}$. If a torque, $\bar{T}$, is exerted on this rotor through the gimbal system as shown, the resultant precession rate, $\dot{\sigma}$, will be at right angles to both the direction of the spin axis of the gyro and the axis of torque application.

Actually there is a small dip angle, ϕ, of the rotor in the direction of the torque as well. For a spherical rotor the magnitude of this angle is given by: $\phi = T/h\omega$. As ω is usually very large, this angle is small by comparison with the total precession angle σ generated in unit time $= T/h$. As long as the torque is oscillatory then ϕ remains small. However, a bias component of torque would eventually cause the rotor to precess 90°. After that the case would simply spin around the rotor, so the gyro would no longer resist the disturbing torque.

A gyroscope differs, for example, from a wheel which is being torqued in that a constant torque produces a constant precession rate whereas a constant torque applied to a wheel would result in a constant angular acceleration. Furthermore, this precession rate of the gyro varies inversely with its total angular momentum. The magnitude of the precession angle then depends on the torque applied, the time during which it acts, and the angular momentum of the gyro spinning mass. For the single control-axis gyro shown, the precession angle is

$$\sigma = \int_0^t \frac{T_i}{h_s}\, dt.$$

The gyro's angular momentum, $\bar{h}$, can be increased either by increasing the angular rate or the moment of inertia of the rotor. The importance of increasing $\bar{h}$ is illustrated in Fig. 5*b* where it is seen that the total precession angle σ produced by a torque impulse, $\bar{T}$, is much smaller for the case of large angular momentum than for a small angular momentum.

An excellent example of a gyroscope is the Earth itself. Here the spin rate is small, amounting to only 1 revolution per 24 hr., but the mass is very large. Therefore, in spite of perturbing torques such as the drag of the sun and the moon on the Earth's equatorial bulge, the precession rate of the Earth's spin axis is very small.

A gyroscope uses inertial space as a reference. In terrestrial navigation we are concerned with navigating with respect to the surface of the Earth. The Earth itself is spinning in inertial space. For example, in Fig. 6 is shown the path followed in inertial space for a simple great circle flight at 900 knots starting at the equator and including both poles. The path in inertial space turns out to be a curved figure "8."

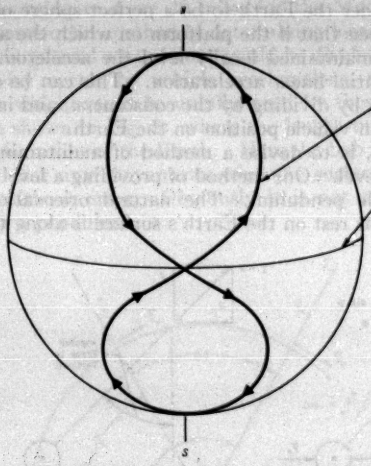


FIG. 6. Great-circle flight path as seen in inertial space.

If an accelerometer is to be maintained locally level and pointing along the vehicle track throughout the flight both Earth rate with respect to inertial space and vehicle rate with respect to the Earth will have to be taken into account.

2. The Schuler Tuning of Horizontal Accelerometers

For determination of position on the Earth's surface two accelerometers, usually mounted with their input axes orthogonal, will be required.

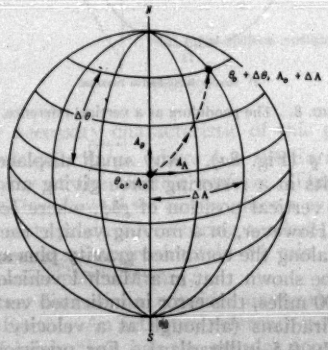


FIG. 7. Inertial guidance on a spherical Earth.

For the moment consider the Earth to be a perfect sphere of radius a (Fig. 7). It can be seen that if the platform on which the accelerometers are mounted is maintained locally level the accelerometers will always measure tangential linear acceleration. This can be converted to angular acceleration by dividing by the constant, a, and integrating twice to give changes in vehicle position on the Earth.

Our first job, then, is to devise a method of maintaining the accelerometer platform level. One method of providing a level reference is the use of a simple pendulum. The natural orientation of the pendulum shaft when at rest on the Earth's surface is along the line of

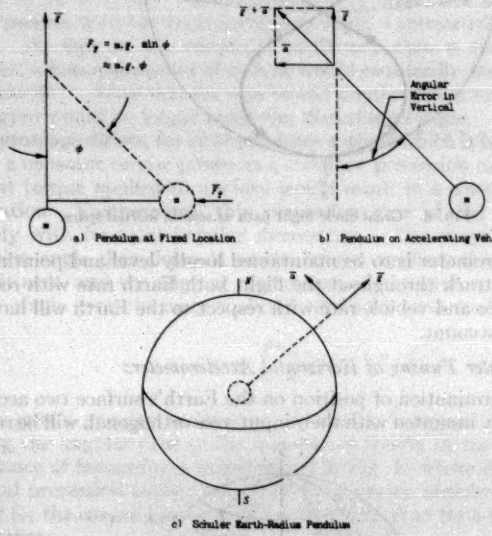

Angular Error in Vertical
a) Pendulum at Fixed Location
b) Pendulum on Accelerating Vehicle
c) Schuler Earth-Radius Pendulum

FIG. 8. The pendulum as a vertical reference.

the gravity vector $\bar{g}$ (Fig. 8a). Any small displacement of the bob from this line results in a restoring force giving an acceleration back toward the locally vertical position of $g\delta\phi$, where $\delta\phi$ is the small displacement angle. However, in a moving vehicle the pendulum would tend to align itself along the combined gravity plus acceleration vector (Fig. 8b). It can be shown that in a Mach 1 vehicle executing a turn with a radius of 1000 miles, this error in indicated vertical would be approximately 5 milliradians (although at a velocity of 180 knots the error would be only 0.5 milliradian). For precision navigation this

error, amounting to some 20 miles for the Mach 1 case, is much too large, so some other method of determining local level must be used.

While in general a pendulum would be displaced from the line of the local gravity vector by vehicle accelerations, there is one particular pendulum which would be uninfluenced by these accelerations. That is the one shown in Fig. 8*c*, where the length of the pendulum shaft is equal to the radius of the Earth. Here the bob is at rest at the Earth's center so that any movements over the surface of the Earth can be followed without error. Obviously it is not practical to attempt to build such a pendulum.

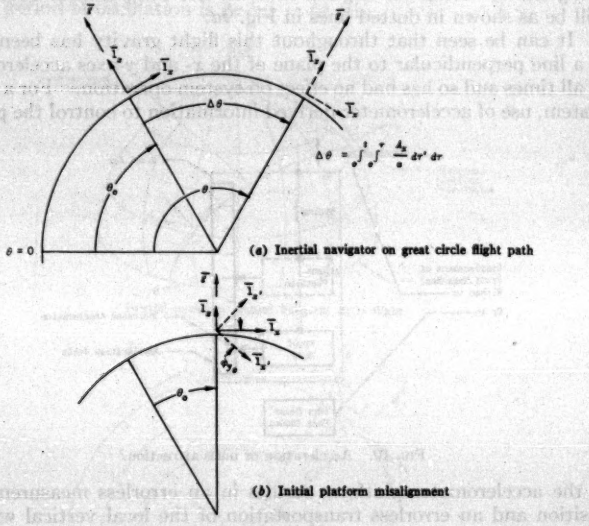

$\Delta\theta = \int_0^t \int_0^{\tau} \frac{A_x}{a}\, d\tau'\, d\tau$
$\theta = 0$
(a) Inertial navigator on great circle flight path
(b) Initial platform misalignment

FIG. 9.

However, the necessary characteristic of this pendulum is that it has a period equal to $2\pi\sqrt{a/g_0}$, where a is the Earth's radius and g_0 is the value of gravity at the Earth's surface. This period is approximately 84 min. Therefore, if we can produce a system in which small angular displacements produce a restoring acceleration equal to $g\delta\phi$ and with a natural period of approximately 84 min., we can mechanize Schuler's Earth-radius pendulum.

To simplify our discussions, consider navigation on a great circle path around the Earth. An arbitrary reference point is taken as $\theta = 0$ and our point of origin then has the angular coordinate θ_0 (Fig. 9*a*).

At this point the inertial navigation system is aligned with its x-axis along the great circle plane, the y-axis perpendicular to the plane, and the z-axis vertically upward to form a right-hand orthogonal set of reference vectors.

Assume that the x-axis accelerometer measures accelerations tangential to the great circle path correctly. These accelerations are divided by the Earth's radius to give angular accelerations and then are doubly integrated without error to give $\Delta\theta$, the change in θ. This information is used without error to change the accelerometer platform position continuously so that it is always tangent to the great circle being flown. After an angular interval $\Delta\theta$, the platform coordinates will be as shown in dotted lines in Fig. 9*a*.

It can be seen that throughout this flight gravity has been acting in a line perpendicular to the plane of the x- and y-axes accelerometers at all times and so has had no effect on system operation. For a perfect system, use of accelerometer-derived information to control the position

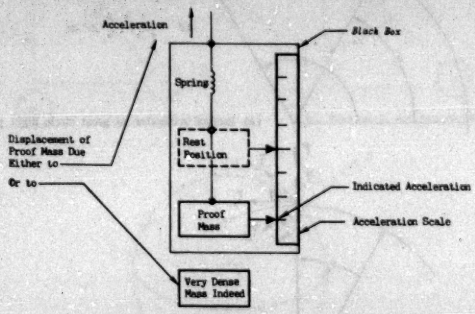


FIG. 10. Acceleration or mass attraction?

of the accelerometer platform results in an errorless measurement of position and an errorless transportation of the local vertical with the vehicle.

Now consider the effect of a small angular error, ϕ_{y_0}, in platform orientation (Fig. 9*b*). System mechanization always assumes that no component of gravity is affecting the accelerometer. As can be seen in this figure, a component of gravity $-g_0\phi_{y_0}$ (considering small angle approximations) is being sensed by the x-axis accelerometer and is interpreted by the computer as a true acceleration. The accelerometer cannot distinguish between actual accelerations and mass attraction (Fig. 10). The resultant system error is derived readily from the differential equations shown below:

$$a\delta\dot{\theta}_y = -g_0\phi_{y_0} - g_0\delta\theta_y$$

or

$$\delta\ddot{\theta}_y + \frac{g_0}{a}\delta\theta_y = -\frac{g_0}{a}\phi_{y_0},$$

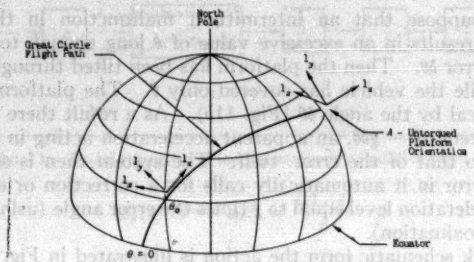
and setting $g_0/a = \omega_0^2$ gives

$$\delta\ddot{\theta}_y + \omega_0^2\delta\theta_y = -\omega_0^2\phi_{y_0}.$$

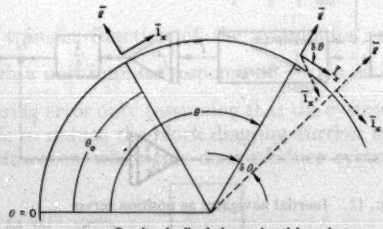
This is the equation for undamped sinusoidal oscillation with the homogeneous solution

$$\theta_y = A \sin(\omega_0 t + \psi).$$

The period of oscillation is $2\pi/\omega_0 \approx 84$ min.



Inertial navigator aligned for great circle flight



Gravity feedback in an inertial navigator

FIG. 11.

We have just shown that a stabilized accelerometer platform, using accelerometer-derived information to control the platform position, has a characteristic period of approximately 84 min. for error inputs. Furthermore, when no error sources are present it can transport the local vertical without error in any vehicle no matter what its speed or acceleration profile may be. This system is a practical mechanization of Schuler's simple Earth-radius pendulum.

This is the historical approach to the analysis of inertial autonavigators. However, for those with an electronics background, an alter-

native approach to the problem appears desirable. This is the simple treatment of the inertial navigator as a position servo.

Visualize the inertial navigator platform lined up for a great-circle flight (Fig. 11*a*). The *x*-axis accelerometer will measure accelerations along this path. The *z*-axis control gyro will keep the accelerometers lined up along the flight path throughout the flight. The *y*-axis gyro, perpendicular to the flight path, would maintain the platform in its origin-level position, as shown at *A*, were it not for the use of accelerometer-derived information on vehicle angular velocity along the great-circle path to supply an additional torque, $\dot{\theta} = V_x/a$. Then for the spherical Earth shown, the platform remains locally level throughout the flight.

Now suppose that an intermittent malfunction in the position computer results in an excessive value of $\dot{\theta}$ long enough to produce a position error $\delta\theta$. Then the platform has been tilted through the angle $\theta + \delta\theta$ while the vehicle has covered only θ. The platform is off the local vertical by the angle $\delta\theta$ (Fig. 11*b*). As a result there is an input to the system of $-g\delta\theta$, an apparent acceleration acting in a direction opposite to that of the error itself. The system then is such that a position error in it automatically calls for a correction or error signal on the acceleration level equal to g times the error angle (using the small angle approximation).

In block schematic form the action is illustrated in Fig. 12. Here

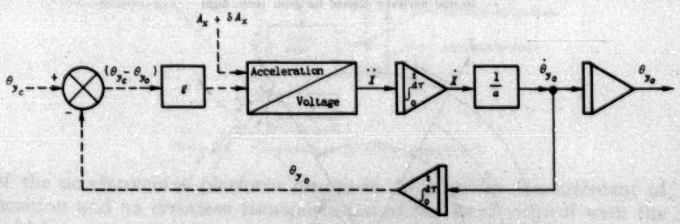


FIG. 12. Inertial navigator as position servo.

θ_{y_c} is the command signal. If the system is operating perfectly then the platform will have been tilted through $\theta_{y_0} = \theta_{y_c}$, so that it will be perpendicular to the local gravity vector with no resultant feedback. However, if for example there is an error in biasing the accelerometer, then there will be an error in the computed value of θ_y, and a resultant error in platform position, giving a feedback signal. From the system block diagram it is a simple matter to derive a transfer function for the system as follows:

$$[(\theta_{y_c} - \theta_{y_0})g + A_x + \delta A_x]\frac{1}{a} = \dot{\theta}_{y_0}.$$

Setting $\theta_{y_b} = \theta_{y_c} + \delta\theta_y$ gives

$$-\delta\theta_y \frac{g}{a} + \frac{A_x}{a} + \delta\frac{A_x}{a} = \ddot{\theta}_y + \delta\ddot{\theta}_y.$$

However $\frac{A_x}{a} = \ddot{\theta}_y$, and, setting $g/a = \omega_0^2$, gives

$$\delta\ddot{\theta}_y + \omega_0^2\delta\theta_y = \frac{\delta A_x}{a}.$$

In Laplace transforms (Fig. 13)

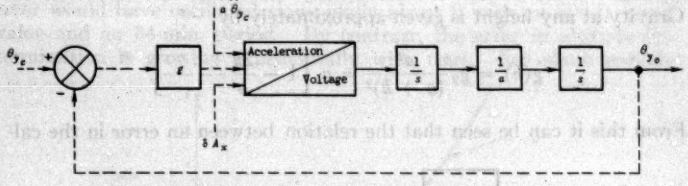


FIG. 13. Transfer function of inertial navigator as position servo.

$$\delta\theta_y(s)\,(s^2 + \omega_0^2) = \frac{\delta A_x(s)}{a}$$

or

$$\delta\theta_y(s) = \frac{\delta A_x(s)}{a}\,\frac{1}{s^2 + \omega_0^2},$$

giving the transfer function of the system for error propagation as $\frac{1}{s^2 + \omega_0^2}$ with a unit impulse response of $\frac{1}{a\omega_0}\sin\omega_0 t$.

Considering error only (assuming that the system is a linear system), it is possible to reduce the block diagram further to the form shown in Fig. 14. However, while this does produce system position error, it

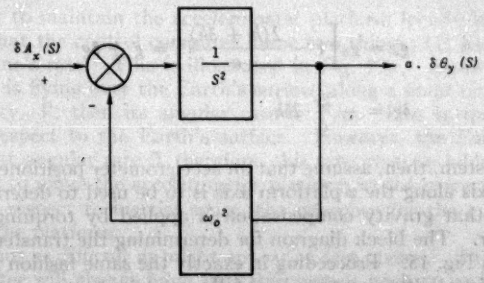


FIG. 14. Simplified block diagram for inertial navigator errors.

omits information on the relation between position error in the computer and platform angular position error, which may be of interest in a number of problems.

3. *The Difficulties in Inertial Altitude Determination*

In the case of position coordinates on the Earth's surface, such as latitude and longitude, gravity feedback is such as to produce an undamped oscillatory error for a given driving function. When an attempt is made to determine the vehicle's height above the Earth's surface through accelerometer measurements, difficulties are encountered. Gravity at any height is given approximately by

$$g(h) = g_0 \frac{a^2}{(a+h)^2} = g_0 \left[1 - \frac{2h}{a} - - - -\right].$$

From this it can be seen that the relation between an error in the cal-

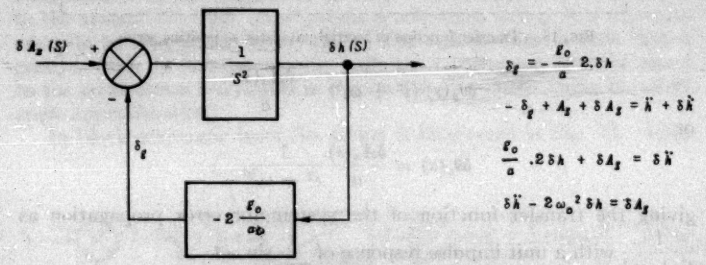


FIG. 15. Feedback loop for inertial altitude determination.

culated value of the apparent acceleration due to gravity and an error in altitude determination is as follows:

$$g_e = g_0 \left[1 - \frac{2(h + \delta h)}{a}\right] = g + \delta g$$

$$\delta g = -\frac{g_0}{a} \cdot 2\delta h.$$

For this system, then, assume that an accelerometer positioned with its sensitive axis along the z-platform axis is to be used to determine altitude, and that gravity compensation is applied by torquing this accelerometer. The block diagram for determining the transfer function is shown in Fig. 15. Proceeding in exactly the same fashion as for the level coordinates, the response is as follows:

$$g(h) - g_0(h + \delta h) + A_z + \delta A_z = \ddot{h} + \delta\ddot{h}; \quad \text{but} \quad A_z = \ddot{h}$$

$$\frac{g_0}{a} \cdot 2\delta h + \delta A_z = \delta\ddot{h} \quad \text{or} \quad \delta\ddot{h} - 2\omega_0^2\delta h = \delta A_z.$$

Considering an initial error in $\dot{h}$ of 1 ft. per sec., gives an error in h thereafter of the form

$$\delta h = \frac{1}{\sqrt{2}\omega_0} \sinh \omega_0 t.$$

Error propagation is shown in Fig. 16. This is contrasted with a level coordinate error which for the same 1-ft.-per-sec. initial velocity error would have oscillated sinusoidally about 0 with an 800-ft. peak value and an 84-min. period. By contrast, the error in altitude determination is growing exponentially with time. For short periods

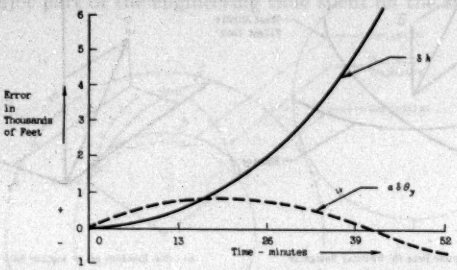


FIG. 16. Comparison of altitude and level error propagation.

this may not be a handicap but for long-time operation stabilization from some other source must be employed if altitude is to be determined inertially.

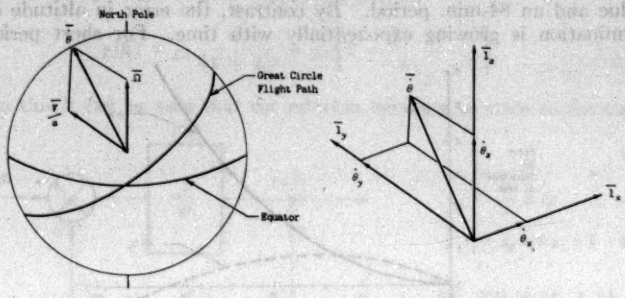
4. Platform as an Angular Rate Resolver

In order to maintain the accelerometer platform locally level, it is necessary that the control computer know two things: (1) Earth rate, and (2) plane's rate. This is illustrated in Fig. 17*a*. Assuming that the aircraft is flying over the Earth's surface along a great-circle route with velocity, $\bar{V}$, then its angular rate is $\bar{V}/a$. This is the plane's rate with respect to the Earth's surface. However, the Earth itself is rotating at angular rate $\bar{\Omega}$, therefore, this rate must be added to the plane's rate to move the accelerometer platform at the correct angular rate $\bar{\theta}$ with respect to inertial space, the reference for the gyroscopes stabilizing the platform.

This is not as difficult as might appear at first glance. The navigation computer will always have available present position and heading

so that the Earth rate components can be calculated for each axis of the accelerometer platform. In addition, present velocity is also known so this determines the angular rate components to be applied about the level platform axes. The problem becomes one of the resolution of a vector angular velocity $\bar{\theta}$ into platform coordinates as shown in Fig. 17*b*. This is a straight-forward problem in analytic geometry.

There is one further complication. In addition to plane's rate and Earth rate there may be an additional rate to be put in about the azimuth axis to keep one of the accelerometer axes pointed in the direction of the pole of the coordinate system. The usual geographic coordinate system with north and south poles is an illustration of this possi-



a) Spatial Angular Rate Of Inertial Navigator b) The Platform as an Angular Rate Resolver

FIG. 17.

bility. Here one accelerometer should be pointing towards the north pole at all times to determine changes in latitude. The orthogonal accelerometer will then measure changes affecting longitude.

C. THE KINDS OF INERTIAL GUIDANCE AND THEIR CHARACTERISTICS

1. Navigation or Gun Aiming

There are two major philosophies followed by newsboys in paper delivery. Very rarely the paper is carried up to your door and deposited exactly on target. More frequently the boy stands on the street and throws the paper as accurately as he is able at your porch or driveway. These philosophies can be followed in the delivery of any payload to a destination. Either a vehicle can be navigated from origin to destination, with course corrections being generated as required, or the entire propulsive force can be applied briefly at the beginning of the trip, with momentum and gravity completing the job of delivery. Of course there is always the possibility that through miscalculation a ballistic projectile will enter a satellite orbit and remain aloft indefinitely (Fig. 18).

In navigation the guidance system continuously computes position throughout the voyage or flight and determines the course to destination from this information. When an inertial guidance system is used for navigation, usually it is the long term error build-up which is important. Factors such as gyro drift cause the dominant errors. Gyro mass unbalance is probably not significant if acceleration levels are low. For the accelerometers, scale factor errors and bias errors cause only small bounded position errors. During this long time of operation, it is necessary that altitude be determined by a pressure altimeter, radar altimeter, or similar device or that these be used to stabilize an inertial method of altitude determination.

In navigation inertial guidance systems long-term gyro drift is the main position error source. Methods of minimizing this error will occupy a major part of the engineering time spent on the system.

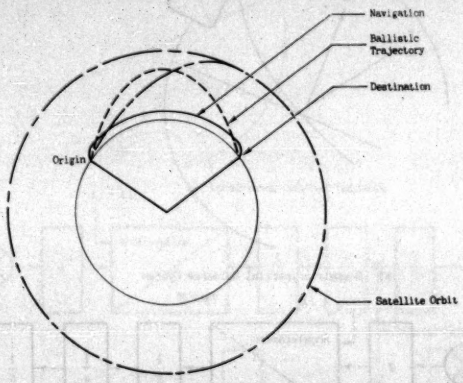


FIG. 18. Navigation, ballistic and satellite paths.

It is possible to treat the vehicle as a projectile from a gun. Here propellant power is maintained only during the initial phase of the trajectory. Position, velocity, and heading at power cut-off determine the destination. The use of guidance has been radically altered in this application, for it is controlling only during the comparatively short initial powered portion of the flight. After cut-off the vehicle follows essentially a ballistic trajectory. Due to this difference in use, the critical components have changed. For gyros, acceleration-sensitive error sources such as mass unbalance are dominant and long-time drift is negligible. For accelerometers, due to the much higher velocity required for a given range using a ballistic trajectory, scale factor errors are important. In many cases, depending on the duration of the powered phase, the instability in altitude determination may be ac-

ceptable, for if powered flight lasts only a few minutes, exponential build-up will not be appreciable. For a gun aiming application then, the engineering emphasis shifts from gyros to accelerometers. The same factors hold for putting the vehicle in a satellite orbit.

2. Forms of Mechanization

There are two design problems of importance in determining mechanization: (1) How is Earth rate to be put into the system? and (2) How is the local vertical to be tracked? Three main forms of mechani-

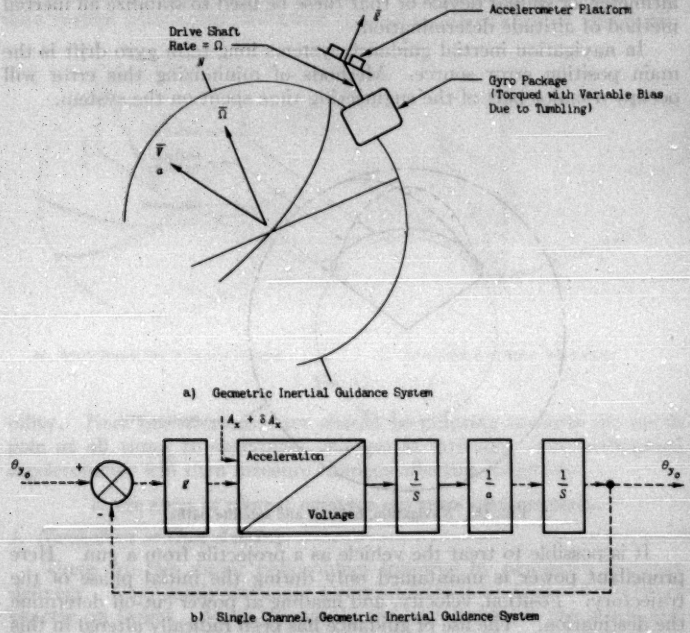


FIG. 19.

zation have been evolved to provide answers to these problems; the choice of each has been influenced by the hardware available at the time of system design and the estimate of the engineers concerned as to the relative difficulties of the engineering problems arising in each mechanization. These forms have been termed Geometric, Semi-analytic, and Analytic. From a descriptive point of view this is a poor choice. Better terminology would be Geometric, Local-Level, and Non-Level systems.

The *Geometric* form of mechanization is shown in crude block form in Fig. 19*a*. Here stabilization comes from a set of gyros maintained fixed in inertial space, so they tumble with respect to the Earth's surface. A drive shaft running parallel with the Earth's North-South axis connects this gyro package with the accelerometer platform. Earth rate is then applied about this axis without reference to the local position indicated by the system. Additional gimbals are inserted between this drive shaft and the accelerometer platform to allow for the inclination of the plane of the great-circle flight path to the Earth's

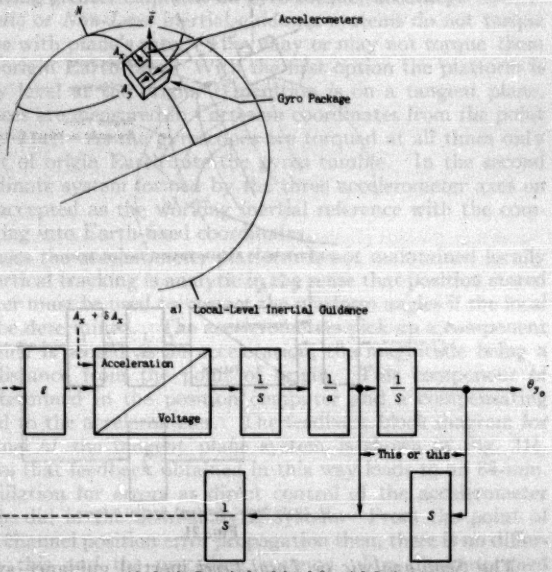


FIG. 20.

equatorial plane and for the vehicle's progress along this great-circle path. Accelerometers are mounted on the platform—one in the track direction, and one in the cross-track direction. Progress along the flight path can be determined by direct measurement of the geometric angle through which the accelerometer platform has been driven from its initial position using the gyro package as a reference. One channel of the system is shown in block form in Fig. 19*b*, where it is seen that no integrations for determining position need occur outside the gravity feedback loop.

The advantage of the system is this: The gyros do not have to be torqued with Earth rate or with plane's rate. Computed position can be determined by direct measurement of gimbal angles. The disadvantages lie in the fact that the gyros tumble, requiring a time and position dependent bias to be generated due to the constantly changing effect of gravity on such things as gyro mass unbalance. In addition the size and weight resulting from piling gimbal upon gimbal as must be done when the system is mounted in a vehicle, can easily get out of hand.

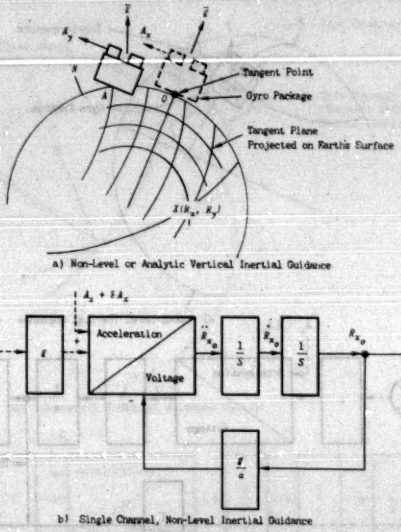


FIG. 21.

The *Semi-analytic* or *Local-Level* inertial guidance system usually puts the gyro reference package and the accelerometer platform together. A rough block diagram of such a system is shown in Fig. 20*a*. In this system a knowledge of present position and platform orientation is required for the determination of the correct Earth rate torques for the gyros. The accelerometer platform is held locally level to track the local vertical. From the accelerometer outputs then it is possible to determine vehicle rate and to use this both to torque the gyros and to apply any required compensating torques to the accelerometers themselves. One channel of system operation is shown in block form in Fig. 20*b*. It should be observed that it is possible to have one of the integrators for the calculation of position outside the gravity feedback

loop. It is possibly this fact which led some people in the past to conclude erroneously that a Semi-analytic system was necessarily long-time unstable.

The advantages of this system are that a local level reference is provided at all times. There are no range limitations and changes in flight plan do not affect the physical alignment of the platform. The gyros do not tumble, so the initial bias will be effective throughout the flight. There is the disadvantage that time-and-position-dependent varying torques must be applied to the gyros and these of considerable magnitude placing greater emphasis on gyro torquer accuracy.

The *Analytic* or *Non-Level* inertial guidance systems do not torque their platforms with plane's rate. They may or may not torque them with point-of-origin Earth rate. With the first option the platform is aligned locally level at the origin. Operation is on a tangent plane, that is, distances are measured in Cartesian coordinates from the point of origin (Fig. 21*a*). As the gyroscopes are torqued at all times only with the point of origin Earth rate the gyros tumble. In the second case the coordinate system formed by the three accelerometer axes on activation is accepted as the working inertial reference with the computer translating into Earth-fixed coordinates.

In both cases the accelerometer platform is not maintained locally level so the vertical tracking is analytic in the sense that position stored in the computer must be used to correct the platform angles if the local vertical is to be determined. The accelerometers pick up a component of gravity which is sensed as an acceleration, the magnitude being a function of distance from the point of origin. This component of gravity is determined in the position computer and a compensating torque applied to the accelerometer. The feedback block diagram for a single channel of the tangent plane system is shown in Fig. 21*b*. Analysis shows that feedback obtained in this way leads to an 84-min. period of oscillation for errors as direct control of the accelerometer platform angle did in the Semi-analytic system. From the point of view of single channel position error propagation then, there is no difference between this and the preceding systems. For the untorqued system, however, auxiliary stabilization of the vertical feedback loop must be employed.

The advantages of the system are this: Torques dependent on plane's rate do not have to be applied to the gyros. This has been exchanged for the simpler problem, over the restricted range allowed by this system, of torquing the accelerometers to compensate for gravity pick-up. The disadvantage is that the gyros tumble, either producing sizeable errors or requiring generation of time-and-position-variable torques with a previous calibration run. In addition a range limitation exists for the tangent plane system, and an additional stabilization problem has been added for the untorqued system.

Electronic "Cat Eye" Snaps the Stars.—The Air Force has described as successful its use of the "Cat Eye" in obtaining previously impossible daylight photographs of planets and stars. Before judging the success of the device, detailed studies were made by Air Research and Development Command (ARDC) scientists at ARDC's Wright Air Development Center (WADC) of several hundred photographs taken during the past 16 months.

A brainchild of Radames K. H. Gebel, of WADC's Aeronautical Research Laboratory, "Cat Eye" is an electronic system that operates on principles similar to television but contains an optical amplifier that affords it a sensitivity of more than 1000 times that of an ordinary TV camera. It is about 10,000 times faster than the best photographic film, and can literally see in the dark.

Coupled with a 10-in. refracting telescope, the improved "Cat Eye" uses the latest WADC findings in light amplification to permit very short photographic exposures during daylight or darkness.

By electronically varying contrasts between light and dark, "Cat Eye" detects very small differences in light which are not apparent to conventional photographic equipment.

Photographs of the moon, planets and other stellar bodies are shot at 1/25th of a second or less, and are equally and often more revealing than conventional telescopic photographs which require exposures up to several minutes.

The brief exposure time permits investigation of the fluctuations of the Earth's atmosphere which gives stars the appearance of twinkling. Similarly, this same atmospheric fluctuation phenomenon adversely affects photographic time exposure—a problem that would be alleviated by "Cat Eye."

Underwater Motor.—A motor designed for use under water—allowing the insulation system of its windings, bearings and magnetic components to be totally immersed—has been announced by General Electric. The "Wet" motor is the first of its type to be designed and produced by an American company. A new insulation system using irradiated polyethylene was the major factor in development.

The anticipated market is as yet only partly explored. It is expected ultimately to include a wide range of applications throughout industry. There are two major uses, at present—as underwater drive motors and integrated pump-motors.

The primary advantage of the "Wet" motor in pumping applications is in high pressure boiler circulating uses, since the need for a pump seal is eliminated. Water from the pump is permitted to circulate through the internal electrical and mechanical components of the motor. A standard pump seal might have to withstand pressure up to 3000 psi. but the "Wet" motor functions efficiently in water under this pressure.

Motors of 250 and 350 hp. are currently being produced for boiler circulating pump applications.

In another application, requiring immersion of the motor in salt water, a 50-hp. "Wet" motor was mounted on the rudder of a ship to provide maneuverability at low speeds.

The potential uses include deep wells which would be under high pressure; a faster and simpler way of dewatering mines; sewage, and nuclear applications. The "Wet" motor eliminates breakdown suffered by conventional insulation systems in such applications.

STRESSES IN ROTATING THIN PLATES WITH CURVILINEAR BOUNDARIES

BY

G. M. ARKILIC [1]

ABSTRACT

The problem of curvilinear plates rotating at constant speed about an axis in the plane of the plate is considered in this paper. Stress components are expressed in terms of coordinates and in terms of certain potential functions. It is shown that by suitable construction of these functions the stress components can be easily determined.

NOMENCLATURE

The following nomenclature is used in the paper:

x, y = rectangular coordinates
ξ, η = orthogonal curvilinear coordinates
ω = angular velocity
$\sigma_x, \sigma_y, \tau_{xy}$ = stress components
$\sigma_\xi, \sigma_\eta, \tau_{\xi\eta}$ = stress components in curvilinear coordinates
$r^2 = x^2 + y^2$
$z = x + iy, \quad \bar{z} = x - iy$
Re, Im = real and imaginary part of
Ω_0 = a harmonic function
$\Omega = \sigma_x + \sigma_y$
∇^2 = the Laplace's operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
Ψ, Φ, F_1, G_1 = conjugate harmonic functions
$F = 8F_1$ and $G = 8G_1$
θ = angle between (ξ = constant) and x-axis
J = stretch ratio
c, λ = real positive constants
n = a positive integer
$P = 1 + \lambda^2(n+1)^2, \quad P_1 = 1 + \lambda^2(n+1)$
$Q = 2\lambda(n+1), \quad Q_1 = \lambda(n+2)$

1. INTRODUCTION

In the absence of body forces, expressing the stress components in terms of harmonic functions, Sen (1)[2] developed a method which was convenient for the solution of plates with curvilinear boundaries.

Treating inertial forces as body forces and developing this method further Gupta (2) solved the problem of steady rotating plates about axes which are normal to the plane of the plates.

In this paper it is shown that with a proper choice of potential

[1] Department of Engineering Mechanics, The Pennsylvania State University, University Park, Pa. Presently with the George Washington University, Washington, D. C.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

function the problem of curvilinear plates rotating at constant speed about an axis in the plane of the plates can also be solved.

The method is illustrated by solving the boundary stresses in circular plates, and plates in the forms of two cusped-epicycloid, dumb-bell and cog-wheel, as shown in Figs. 1, 2, and 4.

Hodge (3) using suitable stress functions obtained solutions for rotating circular and triangular plates. To solve problems with curvilinear boundaries by this method would be very difficult. However, with the method employed in this paper once the solution for a regular curvilinear polygon of n-sides ($n \geq 2$) is obtained, then changing n and the other constants involved, one can easily obtain the solutions for the above mentioned boundaries.

2. THEORY

Let x–y plane be the middle plane of the plate and y be the axis of rotation and ω be the constant angular velocity of rotation.

Assume that the plate is in a state of generalized plane stress, and the inertia forces are the only body forces, then we have the equilibrium equation (4)

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = -\rho\omega^2 x$$
$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0, \tag{1}$$

and the compatibility equation

$$\nabla^2 \Omega = -(1 + \nu)\rho\omega^2, \tag{2}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \Omega = \sigma_x + \sigma_y. \tag{3}$$

Let Ω_0 be a harmonic function then a solution to Eq. 2 may be taken in the form

$$\Omega = \Omega_0 - \tfrac{1}{4}(1 + \nu)\rho\omega^2 r^2. \tag{4}$$

Differentiating Eq. 1 and using Eqs. 3 and 4,

$$\nabla^2[\sigma_x + \tfrac{1}{8}(3 + \nu)\rho\omega^2 r^2] + \frac{\partial^2 \Omega_0}{\partial x^2} = 0$$
$$\nabla^2[\sigma_y - \tfrac{1}{8}(1 - \nu)\rho\omega^2 r^2] + \frac{\partial^2 \Omega_0}{\partial y^2} = 0 \tag{5}$$
$$\nabla^2 \tau_{xy} + \frac{\partial^2 \Omega_0}{\partial x \partial y} = 0.$$

The solution of Eqs. 1, satisfying the consistency relations (5), may be taken as,

$$\sigma_x = \frac{y}{4}\frac{\partial \Omega_0}{\partial y} - \frac{x}{4}\frac{\partial \Omega_0}{\partial x} + \frac{\Omega_0}{2} - \tfrac{1}{8}(3+\nu)\rho\omega^2 r^2 - \tfrac{1}{8}(1-\nu)\rho\omega^2(x^2-y^2) + \Psi$$

$$\sigma_y = -\frac{y}{4}\frac{\partial \Omega_0}{\partial y} + \frac{x}{4}\frac{\partial \Omega_0}{\partial x} + \frac{\Omega_0}{2} + \tfrac{1}{8}(1-\nu)\rho\omega^2 r^2 + \tfrac{1}{8}(1-\nu)\rho\omega^2(x^2-y^2) - \Psi \quad (6)$$

$$\tau_{xy} = -\frac{x}{4}\frac{\partial \Omega_0}{dy} - \frac{y}{4}\frac{\partial \Omega_0}{\partial x} + \Phi,$$

where Ψ and Φ are two conjugate harmonic functions.

Let the curvilinear coordinate be represented by $z = f(\zeta)$ in which $\zeta = \xi + i\eta$ then (5),

$$\cos\theta = \frac{1}{J}\frac{\partial y}{\partial \eta} = \frac{1}{J}\frac{\partial x}{\partial \xi}, \qquad \sin\theta = \frac{1}{J}\frac{\partial y}{\partial \xi} = -\frac{1}{J}\frac{\partial x}{\partial \eta}$$

$$\cos\theta\frac{\partial}{\partial y} - \sin\theta\frac{\partial}{\partial x} = \frac{1}{J}\frac{\partial}{\partial \eta}, \quad \sin\theta\frac{\partial}{\partial y} + \cos\theta\frac{\partial}{\partial x} = \frac{1}{J}\frac{\partial}{\partial \xi} \quad (7)$$

$$J^2 = \left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial y}{\partial \xi}\right)^2 = \left(\frac{\partial x}{\partial \eta}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2$$

and

$$\nabla^2 = \frac{1}{J^2}\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right).$$

The stress components in the two coordinates are related by the following equations.

$$\begin{aligned}
\sigma_\xi &= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \\
\sigma_\eta &= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \\
\tau_{\xi\eta} &= (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta).
\end{aligned} \quad (8)$$

Introducing two conjugate harmonic functions F_1, G_1 such that

$$F_1 + iG_1 = -i(\Phi + i\Psi)\left(\frac{\partial x}{\partial \xi} + i\frac{\partial y}{\partial \xi}\right)^2$$

and making use of the relations (7), we obtain

$$\frac{1}{J^2}F_1 = \Psi(\cos^2\theta - \sin^2\theta) + 2\Phi\cos\theta\sin\theta$$
$$\frac{1}{J^2}G_1 = 2\Psi\sin\theta\cos\theta - \Phi(\cos^2\theta - \sin^2\theta). \tag{9}$$

Substituting (6) in (8) and using Eqs. 7 and 9, one obtains,

$$8J^2\sigma_\xi = \frac{\partial z\bar{z}}{\partial\eta}\frac{\partial\Omega_0}{\partial\eta} - \frac{\partial z\bar{z}}{\partial\xi}\frac{\partial\Omega_0}{\partial\xi} + 4J^2\Omega_0 - (1+\nu)J^2\rho\omega^2 z\bar{z}$$
$$- 2\rho\omega^2 z\bar{z}\,\mathrm{Re}\left(\frac{\partial z}{\partial\xi}\right)^2 - (1-\nu)\rho\omega^2\,\mathrm{Re}(z^2)\,\mathrm{Re}\left(\frac{\partial z}{\partial\xi}\right)^2 + F$$
$$8J^2\sigma_\eta = -\frac{\partial z\bar{z}}{\partial\eta}\frac{\partial\Omega_0}{\partial\eta} + \frac{\partial z\bar{z}}{\partial\xi}\frac{\partial\Omega_0}{\partial\xi} + 4J^2\Omega_0 - (1+\nu)J^2\rho\omega^2 z\bar{z} \tag{10}$$
$$+ 2\rho\omega^2 z\bar{z}\,\mathrm{Re}\left(\frac{\partial z}{\partial\xi}\right)^2 + (1-\nu)\rho\omega^2\,\mathrm{Re}(z^2)\,\mathrm{Re}\left(\frac{\partial z}{\partial\xi}\right)^2 - F$$
$$8J^2\tau_{\xi\eta} = -\frac{\partial z\bar{z}}{\partial\eta}\frac{\partial\Omega_0}{\partial\xi} - \frac{\partial z\bar{z}}{\partial\xi}\frac{\partial\Omega_0}{\partial\eta} + 2\rho\omega^2 z\bar{z}\,\mathrm{Im}\left(\frac{\partial z}{\partial\xi}\right)^2$$
$$+ (1-\nu)\rho\omega^2\,\mathrm{Re}(z^2)\,\mathrm{Im}\left(\frac{\partial z}{\partial\xi}\right)^2 - G.$$

These equations may be put in a more suitable form as follows:

$$8J^4\sigma_\xi = \frac{\partial z\bar{z}}{\partial\eta}\frac{\partial}{\partial\eta}(J^2\Omega_0) - \frac{\partial z\bar{z}}{\partial\xi}\frac{\partial}{\partial\xi}(J^2\Omega_0) + 4J^4\Omega_0 - (1+\nu)J^4\rho\omega^2 z\bar{z}$$
$$- 2J^2\rho\omega^2 z\bar{z}\,\mathrm{Re}\left(\frac{\partial z}{\partial\xi}\right)^2 - (1-\nu)J^2\rho\omega^2\,\mathrm{Re}(z^2)\,\mathrm{Re}\left(\frac{\partial z}{\partial\xi}\right)^2$$
$$+ J^2F - \Omega_0\left(\frac{\partial z\bar{z}}{\partial\eta}\frac{\partial J^2}{\partial\eta} - \frac{\partial z\bar{z}}{\partial\xi}\frac{\partial J^2}{\partial\xi}\right)$$
$$8J^4\sigma_\eta = \frac{\partial z\bar{z}}{\partial\xi}\frac{\partial}{\partial\xi}(J^2\Omega_0) - \frac{\partial z\bar{z}}{\partial\eta}\frac{\partial}{\partial\eta}(J^2\Omega_0) + 4J^4\Omega_0$$
$$- (1+\nu)J^4\rho\omega^2 z\bar{z} + (1-\nu)J^2\rho\omega^2\,\mathrm{Re}(z^2)\,\mathrm{Re}\left(\frac{\partial z}{\partial\xi}\right)^2 \tag{11}$$
$$- J^2F + 2J^2\rho\omega^2 z\bar{z}\,\mathrm{Re}\left(\frac{\partial z}{\partial\xi}\right)^2 - \Omega_0\left(\frac{\partial z\bar{z}}{\partial\xi}\frac{\partial J^2}{\partial\xi} - \frac{\partial z\bar{z}}{\partial\eta}\frac{\partial J^2}{\partial\eta}\right)$$

$$8J^4\tau_{\xi\eta} = -\frac{\partial z\bar{z}}{\partial\eta}\frac{\partial}{\partial\xi}(J^2\Omega_0) - \frac{\partial z\bar{z}}{\partial\xi}\frac{\partial}{\partial\eta}(J^2\Omega_0)$$
$$+ 2J^2\rho\omega^2 z\bar{z}\,\mathrm{Im}\left(\frac{\partial z}{\partial\xi}\right)^2 + (1-\nu)J^2\rho\omega^2\,\mathrm{Re}(z^2)\,\mathrm{Im}\left(\frac{\partial z}{\partial\xi}\right)^2$$
$$- J^2G + \Omega_0\left(\frac{\partial z\bar{z}}{\partial\eta}\frac{\partial J^2}{\partial\xi} + \frac{\partial z\bar{z}}{\partial\xi}\frac{\partial J^2}{\partial\eta}\right).$$

From Eqs. 10 or 11, it is seen that the stress components are expressed in terms of the harmonic function Ω_0 and conjugate functions F and G. Assuming a finite form for Ω_0 and utilizing the boundary conditions, it is possible to determine F and G, and consequently the stresses as shown in the appendix.

3. APPLICATION

We shall now calculate the boundary stresses for curvilinear plates of circular, epicycloid, dumb-bell and cog-wheel shapes.

Since $\Omega = \sigma_x + \sigma_y = \sigma_\xi + \sigma_\eta$, and on the boundary the stresses $(\sigma_\xi)_{\xi=0} = 0$, from Eqs. 3 and 4

$$(\sigma_\eta)_{\xi=0} = (\Omega)_{\xi=0} = \left[\Omega_0 - \tfrac{1}{4}(1+\nu)\rho\omega^2 r^2\right]_{\xi=0}$$

and utilizing Eqs. 16 and 20

$$(\sigma_n)_{\xi=0} = \frac{1}{P+Q\cos n\eta}\Bigg[AP + B - \frac{1+\nu}{4}[\lambda Q + (1+\lambda^2)P]\rho\omega^2c^2$$
$$+ \left\{\lambda(n+1)(2A+B+D) - \frac{1+\nu}{4}[2P\lambda + (1+\lambda^2)Q]\rho\omega^2c^2\right\}$$
$$\times\cos n\eta + \left\{D - \frac{1+\nu}{4}\lambda Q\rho\omega^2c^2\right\}\cos 2n\eta\Bigg], \quad (12)$$

where A, B and D, are constants which can be determined as shown in the appendix.

a. Boundary Stresses for Circular Plates

The boundary stresses for a circle rotating with a constant speed about a diameter can be obtained from Eq. 12 by putting $\lambda = 0$. This results in

$$(\sigma_\eta)_{\xi=0} = \tfrac{1}{8}(1-\nu)\rho\omega^2R^2, \quad (13)$$

where $R = c$ which is the radius of the circle.

It should be mentioned here that this formula agrees with the one obtained by Hodge (3), who solved the problem by using a proper stress function.

b. Boundary Stresses for Two Cusped Epicycloids

The curvilinear polygonal boundary becomes a two cusped epicycloid by putting $n = 2$ and $\lambda = 1/3$, Fig. 1. Since

$$(\sigma_\eta)_{\xi=0} = \left[\Omega_0 - \frac{1+\nu}{4}\rho\omega^2\gamma^2\right]_{\xi=0} = A + \tfrac{1}{2}B + \tfrac{1}{2}D(2\cos 2\eta - 1) - \frac{1+\nu}{4}(1 + \lambda^2 + 2\lambda\cos 2\eta)\rho\omega^2c^2$$

we have

$$(\sigma_\eta)_{\xi=0} = \frac{1-\nu}{36}(3 + \cos 2\eta)\rho\omega^2c^2. \qquad (14)$$

It is seen that at the cusps, σ_η is minimum, and its value is $1/2(\sigma_\eta)_{max}$ which occurs at $\eta = 0$ or $\eta = \pi$.

c. Boundary Stresses for Dumb-Bell Shaped Plates

When $n = 2$ and $\lambda < 1/3$ the polygon has the shape of a dumb-bell, Fig. 2. For various values of λ the values of stresses at the boundary are obtained from Eq. 12 and are shown in Fig. 3.

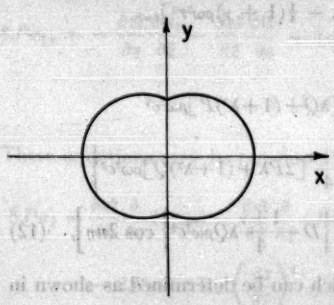


FIG. 1.

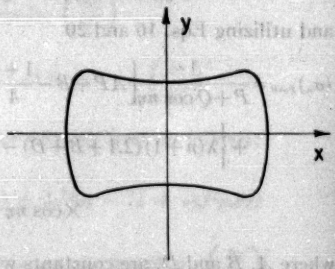


FIG. 2.

d. Boundary Stresses for a Cog-Wheel

Another form that the curvilinear polygon may take is that of a cog-wheel shown in Fig. 4. The stresses at the boundary can be obtained from Eq. 12. The variation of stresses with λ are shown graphically in Fig. 5 for a cog-wheel of $n = 20$ teeth.

4. CONCLUSIONS AND SUMMARY

This paper gives the solution for the boundary stresses in thin curvilinear plates rotating with a constant velocity about an axis in the plane of the plate. The plates considered have at least two axes

of symmetry one of which is assumed to be the axis of rotation as shown in Figs. 1, 2 and 4.

It is found that stresses at the boundary in the plates analyzed (rotating about axes in the plane of the plate) have smaller magnitudes than curvilinear plates, rotating about axes which are perpendicular to the planes of the plates.

For circular and two-cusped epicycloidal shapes the magnitudes of the stresses at the boundary are found to be half of the magnitude for similar shapes rotating about perpendicular axes.

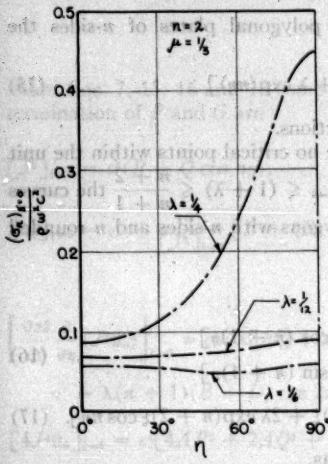


FIG. 3.

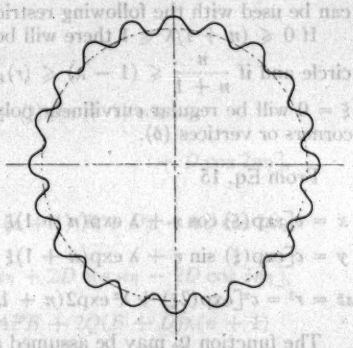


FIG. 4.

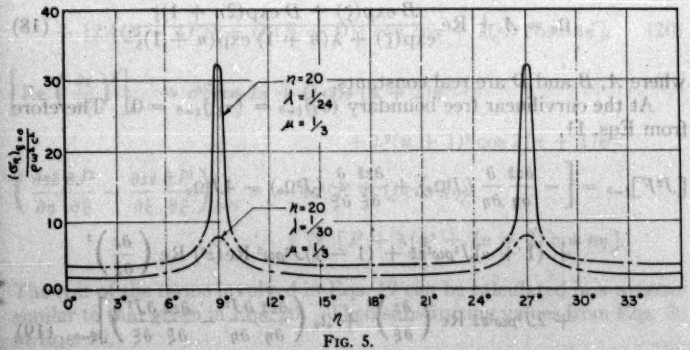


FIG. 5.

Acknowledgments

The author wishes to express his appreciation to Professor Joseph Marin, Head, Department of Engineering Mechanics, The Pennsylvania State University, for his valuable suggestions and also to Mr. John W. Weber for carrying out the numerical work.

APPENDIX

For the solution of curvilinear polygonal plates of n-sides the transformation

$$z = c \exp(\zeta)[1 + \lambda \exp(n\eta)] \tag{15}$$

can be used with the following restrictions.

If $0 \leqslant (n+1)\lambda \leqslant 1$ there will be no critical points within the unit circle and if $\dfrac{n}{n+1} \leqslant (1-\lambda) \leqslant (r)_{\xi=0} \leqslant (1+\lambda) \leqslant \dfrac{n+2}{n+1}$ the curves $\xi = 0$ will be regular curvilinear polygons with n-sides and n rounded corners or vertices (6).

From Eq. 15

$$\begin{aligned} x &= c[\exp(\xi)\cos\eta + \lambda \exp(n+1)\xi \cos(n+1)\eta] \\ y &= c[\exp(\xi)\sin\eta + \lambda \exp(n+1)\xi \sin(n+1)\eta] \end{aligned} \tag{16}$$

$$z\bar{z} = r^2 = c^2[\exp(2\xi) + \lambda^2 \exp 2(n+1)\xi + 2\lambda \exp(n+2)\xi \cos n\eta]. \tag{17}$$

The function Ω_0 may be assumed as,

$$\Omega_0 = A + \operatorname{Re} \frac{B \exp(\zeta) + D \exp(2n+1)\zeta}{\exp(\zeta) + \lambda(n+1)\exp(n+1)\zeta}, \tag{18}$$

where A, B and D are real constants.

At the curvilinear free boundary $(\sigma_\xi)_{\xi=0} = (\tau_{\xi\eta})_{\xi=0} = 0$. Therefore from Eqs. 11,

$$\begin{aligned} [J^2 F]_{\xi=0} = \Bigg[&- \frac{\partial z\bar{z}}{\partial \eta}\frac{\partial}{\partial \eta}(J^2\Omega_0) + \frac{\partial z\bar{z}}{\partial \xi}\frac{\partial}{\partial \xi}(J^2\Omega_0) - 4J^4\Omega_0 \\ &+ (1+\nu)J^4\rho\omega^2 z\bar{z} + (1-\nu)J^2\rho\omega^2 \operatorname{Re}(z^2) \operatorname{Re}\left(\frac{\partial z}{\partial \xi}\right)^2 \\ &+ 2J^2\rho\omega^2 z\bar{z} \operatorname{Re}\left(\frac{\partial z}{\partial \xi}\right)^2 + \Omega_0\left(\frac{\partial z\bar{z}}{\partial \eta}\frac{\partial J^2}{\partial \eta} - \frac{\partial z\bar{z}}{\partial \xi}\frac{\partial J^2}{\partial \xi}\right)\Bigg]_{\xi=0} \end{aligned} \tag{19}$$

$$[J^3G]_{\xi=0} = \left[-\frac{\partial z\bar{z}}{\partial \eta}\frac{\partial}{\partial \xi}(J^3\Omega_0) - \frac{\partial z\bar{z}}{\partial \xi}\frac{\partial}{\partial \eta}(J^3\Omega_0) + 2J^3\rho\omega^2 z\bar{z}\,\mathrm{Im}\left(\frac{\partial z}{\partial \xi}\right)^2 + (1-\nu)J^3\rho\omega^2\,\mathrm{Re}(z^2)\,\mathrm{Im}\left(\frac{\partial z}{\partial \xi}\right)^2 + \Omega_0\left(\frac{\partial z\bar{z}}{\partial \eta}\frac{\partial J^3}{\partial \xi} + \frac{\partial z\bar{z}}{\partial \xi}\frac{\partial J^3}{\partial \eta}\right)\right]_{\xi=0}.$$

From Eqs. 7, 15, 16 and 18 some of the expressions needed for the determination of F and G are

$$[J^3]_{\xi=0} = c^3(P + Q\cos n\eta),$$

$$[\Omega_0]_{\xi=0} = A + \frac{c^3}{[J^3]_{\xi=0}}[B + (B+D)\lambda(n+1)\cos n\eta + D\cos 2n\eta],$$

$$\left[\frac{\partial z\bar{z}}{\partial \eta}\frac{\partial}{\partial \eta}(J^3\Omega_0)\right]_{\xi=0} = c^4n^2\lambda[AQ + \lambda(n+1)(B+D) - AQ\cos 2n\eta - \lambda(n+1)(B+D)\cos 2n\eta + 2D\cos n\eta - 2D\cos 3n\eta],$$

$$[4J^4\Omega_0]_{\xi=0} = c^4[4AP^2 + 2AQ^2 + 4PB + 2Q(B+D)\lambda(n+1) + \{8APQ + 2P(B+D)Q + 4QB + 2QD\}\cos n\eta + \{2AQ^2 + 4PD + Q^2(B+D)\}\cos 2n\eta + 2QD\cos 3n\eta], \quad (20)$$

$$\left[\mathrm{Re}\left(\frac{\partial z}{\partial \xi}\right)^2\right]_{\xi=0} = c^2[\cos 2\eta + Q\cos(n+2)\eta + \lambda^2(n+1)^2\cos 2(n+1)\eta],$$

$$\left(\frac{\partial z\bar{z}}{\partial \eta}\frac{\partial J^3}{\partial \xi} - \frac{\partial z\bar{z}}{\partial \xi}\frac{\partial J^3}{\partial \xi}\right)_{\xi=0} = -4c^4(P + Q\cos n\eta) \times [P + \lambda(n^2+2n+2)\cos n\eta].$$

The rest of the terms involved in Eqs. 19 can be calculated in a manner similar to that given in Eqs. 20. Then substituting values from Eqs. 20 in Eqs. 19,

$$[J^3(F+iG)]_{t=0} = (1-\nu)\rho\omega^2c^6\left\{\frac{\lambda^2}{16}Q^3\exp i(5n+4)\eta\right.$$

$$+\left[\frac{\lambda^2}{8}PQ^2+\frac{\lambda}{8}Q^3+\frac{\lambda^2}{4}Q^3\right]\exp i(4n+4)\eta$$

$$+\left[P\left(\frac{\lambda}{4}Q^2+\frac{\lambda}{2}Q\right)+\tfrac{1}{16}Q^3+\frac{\lambda}{2}Q^2+\frac{\lambda^2}{4}Q+\frac{\lambda}{16}Q^3\right]$$

$$\times\exp i(3n+4)\eta+\left[P(\tfrac{1}{8}Q^2+\lambda Q+\tfrac{1}{2}\lambda^2)+\frac{\lambda}{2}Q+\tfrac{1}{4}Q^2\right.$$

$$+\frac{\lambda}{8}Q^3+\frac{\lambda^2}{4}Q^2\Big]\exp i(2n+4)\eta+\left[P(\lambda+\tfrac{1}{2}Q)+\tfrac{1}{4}Q\right.$$

$$+\tfrac{1}{16}Q^3+\frac{\lambda}{2}Q^2+\frac{\lambda^2}{4}Q\Big]\exp i(n+4)\eta+\tfrac{1}{4}Q\exp i(4-n)\eta$$

$$+[\tfrac{1}{2}P+\tfrac{1}{2}Q(\lambda+\tfrac{1}{2}Q)]\exp i4\eta\Big\}+\rho\omega^2c^2\left\{\lambda Q\exp i(2-2n)\eta\right.$$

$$+[2P\lambda+Q(1+\lambda^2)+\lambda Q^2]\exp i(2-n)\eta$$

$$+\left[2P(1+\lambda^2)+Q[2P\lambda+Q(1+\lambda^2)]+2\lambda Q+\frac{\lambda}{4}Q^3\right]$$

$$\times\exp i2\eta+\left[2P(1+\lambda^2)Q+2P\lambda+Q(1+\lambda^2)\right.$$

$$+[2P\lambda+Q(1+\lambda^2)]\frac{Q^2}{4}+2\lambda Q^2\Big]\exp i(n+2)\eta$$

$$+\left[\tfrac{1}{2}P(1+\lambda^2)Q^2+Q[2P\lambda+Q(1+\lambda^2)]+\lambda Q+\frac{\lambda}{2}Q^3\right]$$

$$\times\exp i(2n+2)\eta+\left[[2P\lambda+Q(1+\lambda^2)]\frac{Q^2}{4}+\lambda Q^2\right]$$

$$\times\exp i(3n+2)\eta+\frac{\lambda}{4}Q^3\exp i(4n+2)\eta\Big\}+N_1\cos n\eta$$

$$+N_2\cos 2n\eta+N_3\cos 3n\eta+i(M_1\sin n\eta+M_2\sin 2n\eta$$

$$+M_3\sin 3n\eta)+L_0 \quad (21)$$

where

$$L_0 = c^4\Big\{\Big[[4P_1[1+\lambda^2(n+1)^2] - n^2\lambda Q + Q_1Q(n+2) - [8P^2 + 2Q^2 + 2\lambda Q(n^2+2n+2)]\Big]A + [4P_1 - \tfrac{1}{2}n^2\lambda Q + Q_1{}^2(n+1) - 8P - Q^2 - Q\lambda(n^2+2n+2)]B + [\tfrac{1}{2}QQ_1(3n+2) - \tfrac{1}{2}n^2\lambda Q - Q^2 - Q\lambda(n^2+2n+2)]D + (1+\nu)\rho\omega^2c^2[(P^2+\tfrac{1}{2}Q^2)(1+\lambda^2) + 2\lambda Q] + (1-\nu)\rho\omega^2c^2 \times \Big[P(\tfrac{1}{2} + \lambda Q + \tfrac{1}{8}\lambda^2Q^2) + \frac{Q}{2}\Big(\lambda + \frac{Q}{2} + \frac{\lambda Q^2}{4} + \frac{\lambda^2 Q}{2}\Big)\Big]\Big\}.$$

$$N_1 = c^4\Big\{[2P_1Q(n+2) + 4Q_1 + Q^2(n+1) - 12PQ - 4P\lambda(n^2+2n+2)]A + [P_1Q(n+2) + 4Q_1 - 2PQ - 6Q - 4\lambda(n^2+2n+2)]B + [(3n+2) - 2n^2\lambda + 2Q_1(n+1) - 2PQ - 4Q - 2\lambda(n^2+2n+2)]D + (1+\nu)\rho\omega^2c^2\Big[2\lambda(P^2+\tfrac{1}{2}Q^2) + 2(1+\lambda^2)PQ + \frac{\lambda}{2}Q^2\Big] + (1-\nu)\rho\omega^2c^2\Big[P\Big(\lambda + \frac{Q}{2} + \frac{\lambda Q^2}{4} + \frac{\lambda^2 Q}{2}\Big) + Q\Big(\tfrac{1}{2} + \lambda Q + \frac{\lambda^2Q^2}{8}\Big) + \tfrac{1}{16}Q^3 + \tfrac{1}{4}\lambda^2Q\Big]\Big\}$$

$$N_2 = c^4\Big\{[n^2\lambda Q + QQ_1(n+2) - 2Q^2 - 2\lambda Q(n^2+2n+2)]A + [\tfrac{1}{2}n^2\lambda Q + \tfrac{1}{2}QQ_1(n+2) - Q^2 - \lambda Q(n^2+2n+2)]B + [\tfrac{1}{2}n^2\lambda Q + 4P_1(n+1) + (3n+2) - 8P - Q^2 - \lambda Q(n^2+2n+2)]D + (1+\nu)\rho\omega^2c^2 \times [2\lambda PQ + \tfrac{1}{2}Q^2(1+\lambda^2)] + (1-\nu)\rho\omega^2c^2 \times \Big[P(\tfrac{1}{8}Q^2 + \tfrac{1}{2}\lambda^2) + \frac{Q}{2}\Big(\lambda + \frac{Q}{2} + \frac{\lambda Q^2}{4}\Big) + \frac{\lambda^2 Q}{2}\Big]\Big\} \quad (22)$$

$$N_3 = c^4\Big\{[2n^2\lambda + 2Q_1(n+1) - 2Q - 2\lambda(n^2+2n+2)]D + \tfrac{1}{2}(1+\nu)\rho\omega^2c^2\lambda^2Q^2 + (1-\nu)\rho\omega^2c^2\frac{Q}{4}\Big(\frac{Q^2}{4} + \lambda^2\Big)\Big\}$$

$$M_1 = c^4\left\{\left[4n\lambda\left[1+\frac{Q^2}{4}(n+1)\right]+2nP_1Q-4nPQ_1\right]A\right.$$
$$+[4n\lambda+nP_1Q-4nQ_1]B+[nQ+nP_1Q+2Q_1+2nQ_1]D$$
$$\left.+(1-\nu)\rho\omega^2c^2\left[P\left(\frac{Q}{2}-\lambda+\frac{\lambda Q^2}{4}-\frac{\lambda^3Q}{2}\right)+\frac{Q^3}{16}-\frac{\lambda^3Q}{4}\right]\right\}$$

$$M_2 = c^4\left\{\left[\tfrac{1}{2}n\lambda Q(3n+2)+4nP_1-\frac{n}{2}QQ_1\right]D\right.$$
$$\left.+(1-\nu)\rho\omega^2c^2\left[P\left(\frac{Q^2}{8}-\frac{\lambda^2}{2}\right)+\frac{Q^2}{4}-\frac{\lambda Q}{2}+\frac{\lambda Q^2}{8}-\frac{\lambda Q^2}{4}\right]\right\}$$

$$M_3 = c^4\left\{nQD+(1-\nu)\rho\omega^2c^2\frac{Q}{4}\left(\frac{Q^2}{4}-\lambda^2\right)\right\}.$$

Now we may assume

$$F+iG = \frac{\sum_{m=1}^{3}A_m\exp(mn+1)\zeta+\sum_{m=-1}^{4}B_m\exp(mn+3)\zeta+\sum_{m=0}^{5}C_m\exp(mn+5)\zeta}{\exp\zeta+\lambda(n+1)\exp(n+1)\zeta}. \tag{23}$$

Hence, from Eq. 23

$$[J^3(F+iG)]_{\zeta=0} = c^2\{\lambda(n+1)A_1+[A_1+\lambda(n+1)A_2]$$
$$\times\exp in\eta+[A_2+\lambda(n+1)A_3]\exp i2n\eta+A_3\exp i3n\eta$$
$$+\lambda(n+1)B_1\exp i(2-2n)\eta+[B_{-1}+\lambda(n+1)B_0]$$
$$\times\exp i(2-n)\eta+[B_0+\lambda(n+1)B_1]\exp i2\eta$$
$$+[B_1+\lambda(n+1)B_2]\exp i(n+2)\eta+[B_2+\lambda(n+1)B_3]$$
$$\times\exp i(2n+2)\eta+[B_3+\lambda(n+1)B_4]\exp i(3n+2)$$
$$+B_4\exp i(4n+2)\eta+C_2\lambda(n+1)\exp i(4-n)\eta$$
$$+[C_0+\lambda(n+1)C_1]\exp i4\eta+[C_1+\lambda(n+1)C_2]$$
$$\times\exp i(n+4)\eta+[C_2+\lambda(n+1)]\exp i(2n+4)\eta$$
$$+[C_3+\lambda(n+1)C_4]\exp i(3n+4)\eta+[C_4+\lambda(n+1)C_5]$$
$$\times\exp i(4n+4)\eta+C_5\exp i(5n+4)\eta\}. \tag{24}$$

From the equality conditions of functions (21) and (24) sufficient number of equations can be obtained to determine the unknown coefficients. The values of these coefficients are

$$A = \frac{\rho\omega^2c^2}{32(n+1)(\lambda Q-2)}\{Q^4 - 6\lambda Q^3 - [(4n^2-4n-14)\lambda^2 + 8(n+3)]Q^2 + [8n^2+76n+72]\lambda Q - 8(n+9) + \nu\{Q^4 - 2\lambda Q^3 - [(4n^2+4n-2)\lambda^2 + 8(n+1)]Q^2 + [8n^2+36n+24]\lambda Q - 24(n+1)\}\}$$

$$B = \frac{\rho\omega^2c^2}{16(n+1)(2-\lambda Q)}\{Q^4 - 6\lambda Q^3 - [(4n^2-4n-8)\lambda^2 + 8(n+3)]Q^2 + [8n^2+68n+60]\lambda Q + 8(n-3) + \nu\{Q^4 - 2\lambda Q^3 - [(4n^2+4n)\lambda^2 + 8(n+1)]Q^2 + [8n^2+28n+20]\lambda Q - 8(n+1)\}\}$$

$$D = \tfrac{1}{4}\rho\omega^2c^2\lambda^2[2n+3+\nu(2n+1)]$$

$$A_1 = \tfrac{1}{16}\rho\omega^2c^4\lambda\{8n(5-n) + 16\lambda^2n^2(2n+7) + 24\lambda^2(3n-1) - 4n\lambda Q(2n^2+20n+19) + 8nQ^2(n+3) - 2\lambda^2Q^2(7n+2)(n+1) + 6Q^2(n+1) + n\lambda Q^3(2n+7) - Q^4(n-1) + \nu\{8n(n-1) + 24(n+1)\lambda^2 + 16(2n^3+5n^2+2n)\lambda^2 - 4\lambda Q(2n^3+4n^2+n) + 2Q^2(4n-3)(n+1) + 2\lambda^2Q^2(2-n)(n+1) + 6\lambda nQ^3 + 2\lambda Q^3(n^2-1) - Q^4(n+1)\}\} \quad (25)$$

$$A_2 = \tfrac{1}{8}\rho\omega^2c^4\lambda^2\{\lambda(n+2)(4n^2+6n)Q - (2n^2+n)Q^2 + (28n^2+40n) + \nu\{\lambda(n+2)(4n^2+2n)Q - (2n^2+4n-2)Q^2 + (4n^2-8n)\}\}$$

$$A_3 = \tfrac{1}{4}\rho\omega^2c^4\lambda^2Q[n^2(3+\nu) + 5n - 2\nu]$$

$$B_{-1} = 2\rho\omega^2c^4\lambda$$

$$B_0 = 2\rho\omega^2c^4[1+\lambda^2+3\lambda^2(n+1)]$$

$$B_1 = 2\rho\omega^2c^4\lambda[1+3(n+1)+3\lambda^2(n+1)+3\lambda^2(n+1)^2]$$

$$B_2 = 2\rho\omega^2c^4\lambda^2(n+1)[3+3(n+1)+3\lambda^2(n+1)+\lambda^2(n+1)^2]$$

$$B_3 = 2\rho\omega^2c^4\lambda^3(n+1)^2[3+(n+1)+\lambda^2(n+1)]$$

$$B_4 = 2\rho\omega^2c^4\lambda^4(n+1)^3$$

$$C_0 = \tfrac{1}{2}(1-\nu)\rho\omega^2c^4$$

$$C_1 = \tfrac{1}{2}(1-\nu)\rho\omega^2c^4\lambda(3n+5)$$

$$C_2 = \tfrac{1}{2}(1-\nu)\rho\omega^2c^4\lambda^2(3n^2+12n+10)$$

$$C_3 = \tfrac{1}{2}(1-\nu)\rho\omega^2c^4\lambda^3(n+1)(n^2+8n+10)$$

$$C_4 = \tfrac{1}{2}(1-\nu)\rho\omega^2c^4\lambda^4(n+1)^2(2n+5)$$

$$C_5 = \tfrac{1}{2}(1-\nu)\rho\omega^2c^4\lambda^5(n+1)^3.$$

REFERENCES

(1) B. SEN, "Direct Determination of Stresses from the Stress Equations in Some Two-Dimensional Problems of Elasticity. Part V. Problems of Curvilinear Boundaries," *Phil. Mag.*, Ser. 7, Vol. 39, p. 992 (1948).

(2) A. M. GUPTA SEN, "Stresses in Certain Thin Elastic Plates Rotating about Normal Axes," *Zeits. für Angewandte Mathematik und Mechanik*, Vol. 37, p. 27, Jan./Feb. (1957).

(3) P. G. HODGE, JR., "Stress Functions for Rotating Plates," *J. Appl. Mech., Trans. ASME*, Vol. 23, No. 2, p. 273 (1956).

(4) S. TIMOSHENKO AND J. N. GOODIER, "Theory of Elasticity," New York, McGraw-Hill Book Co., Inc., Second Edition, Sect. 15, 1951.

(5) E. G. COKER AND L. N. G. FILON, "A Treatise on Photo-Elasticity," New York, Cambridge University Press, pp. 153, 164, 1931.

(6) A. C. STEVENSON, "The Boundary Couples in Thin Plates," *Phil. Mag.*, Ser. 7, Vol. 34, p. 112 (1943).

(7) M. ROTHMAN, "Isolated Force Problems in Two-Dimensional Elasticity (II)," *Quart. J. Mech. Appl. Mathematics*, Vol. 3, p. 469 (1950).

TWO-DIMENSIONAL PERIODIC FLOW OF HEAT IN POLAR COORDINATES

BY

HERBERT REISMANN [1]

ABSTRACT

This paper develops a technique for solving two-dimensional problems of periodic heat flow in polar coordinates. The form of series solution presented permits satisfaction of very general boundary conditions specified as surface temperature distributions which are periodic with respect to time. The method is applied to the solution of the radially symmetric plane problem of heat flow in an infinite region, exterior to a circular boundary, to which a uniformly distributed temperature is applied which varies periodically with respect to time.

NOMENCLATURE

The following nomenclature is used in this paper:

T = temperature
t = time
A = thermal diffusivity
ω = circular frequency
r = radial coordinate
θ = angular coordinate
ρ = dimensionless radial coordinate = λr
λ = parameter = $\sqrt{A\omega}$
i = $\sqrt{-1}$

INTRODUCTION

The problem of one-dimensional periodic heat flow has been discussed extensively in (1)[2] and (2), while the corresponding treatment of the two-dimensional problem is discussed in (3) for the special case of cartesian coordinates only. It is the object of this paper to find a general solution of the problem of two-dimensional periodic heat flow in polar coordinates. The problem under discussion is of interest in the design and analysis of mechanical or structural equipment with circular boundaries subjected to periodically alternating surface temperatures such as encountered in certain types of combustion chambers, cylinder walls, rotors, etc., where it is desired to calculate the internal temperature distribution.

---

[1] Systems Analysis Staff, The Martin Company, Denver, Colo.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

STATEMENT AND SOLUTION OF PROBLEM

The distribution of temperature in a homogeneous, isotropic solid is characterized by the differential Eq. 1

$$\nabla^2 T = A \frac{\partial T}{\partial t}, \tag{1}$$

where A is the thermal diffusivity of the heat conducting material, assumed to be a constant in this paper, and under the assumption of two-dimensional heat flow in polar coordinates,

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}. \tag{2}$$

It should be noted that Eq. 1 is valid only in regions of the conducting medium which are free from heat sources and sinks.

If we prescribe periodic temperature variations of frequency ω on the boundaries, then the temperature in the interior and on the boundary of the region under consideration may be expressed by

$$T(r, \theta, t) = F(r, \theta) \cos \omega t + G(r, \theta) \sin \omega t. \tag{3}$$

Substituting Eq. 3 into Eq. 1, and letting $\lambda^2 = A\omega$ and $\lambda r = \rho$, we obtain

$$\left.\begin{aligned} \nabla_\rho^2 F &= G \\ \nabla_\rho^2 G &= -F \end{aligned}\right\}, \tag{4}$$

where

$$\nabla_\rho^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \theta^2}. \tag{5}$$

To solve the system of partial differential Eqs. 4, we define the complex variable

$$\bar{W} = F + iG \tag{6}$$

and note that

$$\nabla_\rho^2 \bar{W} = \nabla_\rho^2 F + i\nabla_\rho^2 G = G - iF = -i\bar{W}.$$

Thus the solution of the system of Eqs. 4 has been reduced to the solution of the single differential equation

$$\frac{\partial^2 \bar{W}}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \bar{W}}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 \bar{W}}{\partial \theta^2} = -i\bar{W}. \tag{7}$$

We assume a solution in the form

$$\bar{W}(\rho, \theta) = \sum_{n=0}^{\infty} \bar{R}_n \cos n\theta + \sum_{n=0}^{\infty} \bar{R}_n' \sin n\theta, \tag{8}$$

where $\bar{R}_n$ and $\bar{R}_n'$ are complex functions of the radial coordinate ρ only, and, because of the requirement that $\bar{W}(\rho, \theta)$ be single valued, $n = 1, 2, 3, \cdots$. Substitution of the assumed solution, Eq. 8, into the differential Eq. 7 results in

$$\frac{d^2\bar{R}_n}{d\rho^2} + \frac{1}{\rho}\frac{d\bar{R}_n}{d\rho} + \left(i - \frac{n^2}{\rho^2}\right)\bar{R}_n = 0 \tag{9}$$

and an additional equation which is obtained by replacing $\bar{R}_n$ by $\bar{R}_n'$ in Eq. 9.

The above Eq. 9 is the well known Bessel differential equation of order n and its solution is given by

$$\bar{R}_n = i^{-n}\bar{A}_n I_n(\rho i^{-1/2}) + i^n \bar{B}_n K_n(\rho i^{-1/2}), \tag{10}$$

where $\bar{A}_n$ and $\bar{B}_n$ are arbitrary complex constants of integration and I_n and K_n are known as the modified Bessel functions of the first and second kind, respectively, of order n (see (4), chapter VII).

We note the following definitions

$$\left.\begin{aligned} i^{-n}I_n(\rho i^{-1/2}) &= ber_n\,\rho - i\,bei_n\,\rho \\ i^n K_n(\rho i^{-1/2}) &= ker_n\,\rho - i\,kei_n\,\rho \end{aligned}\right\}. \tag{11}$$

In view of the identities, Eqs. 11, we may write Eq. 10 in the following form:

$$\bar{R}_n = \bar{A}_n(ber_n\,\rho - i\,bei_n\,\rho) + \bar{B}_n(ker_n\,\rho - i\,kei_n\,\rho). \tag{12}$$

Let

$$\left.\begin{aligned} \bar{A}_n &= \alpha_n + i\beta_n \\ \bar{B}_n &= \gamma_n + i\delta_n \end{aligned}\right\}, \tag{13}$$

where α_n, β_n, γ_n, and δ_n are real constants. Then

$$\left.\begin{aligned} \bar{R}_n = {} & \alpha_n\,ber_n\,\rho + \beta_n\,bei_n\,\rho + \gamma_n\,ker_n\,\rho + \delta_n\,kei_n\,\rho \\ & + i(\beta_n\,ber_n\,\rho - \alpha_n\,bei_n\,\rho + \delta_n\,ker_n\,\rho - \gamma_n\,kei_n\,\rho) \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} \bar{R}_n' = {} & \alpha_n'\,ber_n\,\rho + \beta_n'\,bei_n\,\rho + \gamma_n'\,ker_n\,\rho + \delta_n'\,kei_n\,\rho \\ & + i(\beta_n'\,ber_n\,\rho - \alpha_n'\,bei_n\,\rho + \delta_n'\,ker_n\,\rho - \gamma_n'\,kei_n\,\rho) \end{aligned}\right\}. \tag{15}$$

Taking real and imaginary parts of $\bar{W}$, we obtain

$$F(\rho, \theta) = \sum_{n=0}^{\infty} (\alpha_n \, ber_n \rho + \beta_n \, bei_n \rho + \gamma_n \, ker_n \rho + \delta_n \, kei_n \rho) \cos n\theta$$

$$\sum_{n=0}^{\infty} (\alpha_n' \, ber_n \rho + \beta_n' \, bei_n \rho + \gamma_n' \, ker_n \rho + \delta_n' \, kei_n \rho) \sin n\theta \quad (16)$$

$$G(\rho, \theta) = \sum_{n=0}^{\infty} (\beta_n \, ber_n \rho - \alpha_n \, bei_n \rho + \delta_n \, ker_n \rho - \gamma_n \, kei_n \rho) \cos n\theta$$

$$\sum_{n=0}^{\infty} (\beta_n' \, ber_n \rho - \alpha_n' \, bei_n \rho + \delta_n' \, ker_n \rho - \gamma_n' \, kei_n \rho) \sin n\theta. \quad (17)$$

The functions ber_n, bei_n, ker_n, and kei_n play an important role in the theory of alternating electric currents and are discussed from that point of view in (5). They have also been found useful in the theory of circular plates resting on an elastic foundation, as shown in (6). They have been tabulated, and some of their properties relevant to this investigation are given in (4) and (7).

Because of the form of the solution as exhibited by Eq. 3 in conjunction with Eqs. 16 and 17, very general boundary conditions may be satisfied. Considering, for instance, the region of the plane bounded by two concentric circles of radii $\rho = \xi$ and $\rho = \eta$, we may prescribe the following conditions on the boundary:

$$\left.\begin{aligned} T(\xi, \theta, t) &= F(\xi, \theta) \cos \omega t + G(\xi, \theta) \sin \omega t \\ T(\eta, \theta, t) &= F(\eta, \theta) \cos \omega t + G(\eta, \theta) \sin \omega t \end{aligned}\right\}. \quad (18)$$

Assuming that the character of the functions $F(\xi, \theta)$, $G(\xi, \theta)$, $F(\eta, \theta)$, and $G(\eta, \theta)$ is such as to permit their representation by Fourier series, we have

$$\left.\begin{aligned} F(\xi, \theta) &= \sum_{n=0}^{\infty} a_n \cos n\theta + a_n' \sin n\theta \\ G(\xi, \theta) &= \sum_{n=0}^{\infty} b_n \cos n\theta + b_n' \sin n\theta \\ F(\eta, \theta) &= \sum_{n=0}^{\infty} c_n \cos n\theta + c_n' \sin n\theta \\ G(\eta, \theta) &= \sum_{n=0}^{\infty} d_n \cos n\theta + d_n' \sin n\theta \end{aligned}\right\}, \quad (19)$$

where a_n, b_n, c_n, d_n, a_n', b_n', c_n', and d_n' are the known Fourier coefficients.

Application of the boundary conditions, Eqs. 19, to the solution, Eqs. 16 and 17 results in identities in θ giving rise to a system of eight independent linear algebraic equations in eight unknowns:

$$\begin{bmatrix} a_n \\ a_n' \\ b_n \\ b_n' \end{bmatrix} = \begin{bmatrix} \alpha_n & \beta_n & \gamma_n & \delta_n \\ \alpha_n' & \beta_n' & \gamma_n' & \delta_n' \\ \beta_n & -\alpha_n & \delta_n & -\gamma_n \\ \beta_n' & -\alpha_n' & \delta_n' & -\gamma_n' \end{bmatrix} \begin{bmatrix} ber_n\, \xi \\ bei_n\, \xi \\ ker_n\, \xi \\ kei_n\, \xi \end{bmatrix} \tag{20a}$$

$$\begin{bmatrix} c_n \\ c_n' \\ d_n \\ d_n' \end{bmatrix} = \begin{bmatrix} \alpha_n & \beta_n & \gamma_n & \delta_n \\ \alpha_n' & \beta_n' & \gamma_n' & \delta_n' \\ \beta_n & -\alpha_n & \delta_n & -\gamma_n \\ \beta_n' & -\alpha_n' & \delta_n' & -\gamma_n' \end{bmatrix} \begin{bmatrix} ber_n\, \eta \\ bei_n\, \eta \\ ker_n\, \eta \\ kei_n\, \eta \end{bmatrix} \tag{20b}$$

$$(n = 0, 1, 2, 3, \cdots).$$

The solution of Eqs. 20 determines the constants of integration of the problem, and thus the temperature field in the interior of the region will be uniquely defined.

In order to find the maximum temperature T_{max} occurring at time t_1 at a point in the conducting medium, we differentiate Eq. 3 with respect to t and set

$$\frac{\partial T}{\partial t} = 0. \tag{21}$$

Solving the resulting equation, we obtain

$$\tan \omega t_1 = \frac{G(\rho, \theta)}{F(\rho, \theta)}. \tag{22}$$

Evaluating the temperature T at time t_1 we obtain from Eq. 3

$$T_{max} = \sqrt{F^2 + G^2}. \tag{23}$$

Since ωt_1 is the phase angle and $|T_{max}|$ is the amplitude of the temperature oscillation, we can write Eq. 3 in the following form:

$$T = T_{max} \cos \omega(t - t_1). \tag{24}$$

EXAMPLE

We shall now consider the following problem: Two-dimensional radial heat flow occurs in an infinite region exterior to a circular boundary of radius a. The temperature is uniformly distributed at $r = a$

and varies periodically with respect to time with frequency ω and amplitude T_0. It is required to find the temperature field for $r > a$.

Because of radial symmetry, the solution will be independent of the angular coordinate θ. Thus we have a temperature field given by

$$T(\rho) = F(\rho) \cos \omega t + G(\rho) \sin \omega t, \tag{25}$$

where

$$\left.\begin{aligned} F(\rho) &= C_1 \, ker\, \rho + C_2 \, kei\, \rho + C_3 \, ber\, \rho + C_4 \, bei\, \rho \\ G(\rho) &= C_2 \, ker\, \rho - C_1 \, kei\, \rho + C_4 \, ber\, \rho - C_3 \, bei\, \rho \end{aligned}\right\}. \tag{26}$$

Let $\lambda a = \xi$. The appropriate boundary conditions are

$$T(\xi) = T_0 \cos \omega t \tag{27}$$

$$\lim_{\rho \to \infty} T(\rho) = 0. \tag{28}$$

Satisfaction of Eq. 28 requires that $C_3 = C_4 = 0$. Application of Eq. 27 to the solution, Eq. 25, results in

$$\left.\begin{aligned} F(\xi) &= C_1 \, ker\, \xi + C_2 \, kei\, \xi = T_0 \\ G(\xi) &= C_2 \, ker\, \xi - C_1 \, kei\, \xi = 0 \end{aligned}\right\}. \tag{29}$$

Solving,

$$\left.\begin{aligned} C_1 &= \frac{T_0 \, ker\, \xi}{ker^2\, \xi + kei^2\, \xi} \\ C_2 &= \frac{T_0 \, kei\, \xi}{ker^2\, \xi + kei^2\, \xi} \end{aligned}\right\}. \tag{30}$$

Thus,

$$\left.\begin{aligned} F(\rho) &= \frac{T_0 (ker\, \xi \, ker\, \rho + kei\, \xi \, kei\, \rho)}{ker^2\, \xi + kei^2\, \xi} \\ G(\rho) &= \frac{T_0 (kei\, \xi \, ker\, \rho - ker\, \xi \, kei\, \rho)}{ker^2\, \xi + kei^2\, \xi} \end{aligned}\right\}. \tag{31}$$

Substituting Eqs. 31 into Eqs. 22 and 23, respectively, we obtain

$$\omega t_1 = \arctan \left[\frac{kei\, \xi \, ker\, \rho - ker\, \xi \, kei\, \rho}{ker\, \xi \, ker\, \rho + kei\, \xi \, kei\, \rho} \right] \tag{32}$$

and

$$\frac{T_{max}}{T_0} = \sqrt{\frac{ker^2\, \rho + kei^2\, \rho}{ker^2\, \xi + kei^2\, \xi}} \tag{33}$$

Dimensionless plots of Eqs. 32 and 33 are shown in Figs. 1 and 2, respectively.

FIG. 1. Local peak temperature *versus* radial coordinate ($\xi = 0.5$).

FIG. 2. Phase angle *versus* radial coordinate ($\xi = 0.5$).

CONCLUSION

It may be concluded that the method presented in this paper permits the evaluation of the time-dependent temperature field of two-dimensional regions bounded by concentric circles. For this class of problems, the conditions on a circular boundary may be specified by an arbitrary spatial distribution of temperature which is harmonic with respect to time.

It is interesting to note that the radially symmetric example worked in this paper displays the characteristic features of the corresponding problem in linear heat flow (1, 2). In particular, there is a rapid decay of the local peak temperature and a linear increase of the phase angle with increasing penetration into the interior of the solid as shown in Figs. 1 and 2.

REFERENCES

(1) H. S. CARSLAW AND J. C. JAEGER, "Conduction of Heat in Solids," London, Oxford University Press, 1947.

(2) L. R. INGERSOLL, O. J. ZOBEL AND A. C. INGERSOLL, "Heat Conduction," New York, McGraw-Hill Book Co., Inc., 1948.

(3) P. LARDY, "The Two-Dimensional Problem in the Case of Periodically Altering Temperature Effects," *Publications of the International Association for Bridge and Structural Engineering*, Vol. 12, pp. 201–220, Zurich, 1952.

(4) N. W. MCLACHLAN, "Bessel Functions for Engineers," London, Oxford University Press, 1934.

(5) H. B. DWIGHT, "Bessel Functions for Alternating Current Problems," *Trans. A.I.E.E.*, Vol. 48, pp. 812–820 (1929).

(6) HERBERT REISMANN, "Bending of Circular and Ring-Shaped Plates on an Elastic Foundation," *J. Appl. Mech.*, Vol. 21, pp. 45–51 (1954).

(7) H. B. DWIGHT, "Tables of Integrals and Other Mathematical Data," Revised Edition, New York, The Macmillan Company, 1947.

V.2
195

THE FRANKLIN INSTITUTE

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 260 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

RECENT ADDITIONS

AERONAUTICS

CONWAY, H. G. Landing Gear Design. 1958.

KÁRMÁN, THEODORE VON. Collected Works. Vols. 1–4. 1956.

ARCHITECTURE AND BUILDING

GAY, CHARLES MERRICK ET AL. Materials and Methods of Architectural Construction. Ed. 3. 1958.

ASTRONOMY

AMBARTSUMIAN, VIKTOR AMAZASPOVICH, ED. Theoretical Astrophysics. 1958.

AUSTRALIA. COUNCIL FOR SCIENTIFIC AND INDUSTRIAL RESEARCH. RADIOPHYSICS LABORATORY. Symposium on Radio Astronomy. 1957.

WERNER, HELMUT. From the Aratus Globe to the Zeiss Planetarium. 1957.

BIBLIOGRAPHY

U. S. DEPARTMENT OF AGRICULTURE. Bibliography of Forest Disease Research in the Department of Agriculture. 1957.

CHEMISTRY AND CHEMICAL TECHNOLOGY

CHEMICAL SOCIETY. LONDON. Current Trends in Heterocyclic Chemistry. 1958.

COOK, MELVIN A. The Science of High Explosives. 1958.

GILLAM, A. E. AND STERN, E. S. An Introduction to Electronic Absorption Spectroscopy in Organic Chemistry. 1958.

GOEHRING, MARGOT. Ergebnisse und Probleme der Chemie der Schwefelstickstoffverbindungen. 1957.

HUGGINS, MAURICE LOVAL. Physical Chemistry of High Polymers. 1958.

RUDNER, MERRITT ALLEN. Fluorocarbons. 1958.

WEBBER, R. B. The Book of pH. 1957.

DIRECTIONARIES

Dictionary of Physics and Allied Sciences. 1958.

ELECTRICITY AND ELECTRICAL ENGINEERING

BENHAM, W. E. AND HARRIS, T. A. The Ultra High Frequency Performance of Receiving Tubes. 1957.

BROOKLYN, POLYTECHNIC INSTITUTE. MICROWAVE RESEARCH INSTITUTE. Proceedings of the Symposium on the Role of Solid State Phenomena in Electric Circuits. 1957.

PARTRIDGE, GORDON R. Principles of Electronic Instruments. 1958.

SOLODOVNIKOV, V. V.; TOPCHEEV, YU. I. AND KRUTIKOVA, S. V. Procedure for Obtaining Transient Response from Frequency Response with Tables and Nomographs. 1958.

ZWORYKIN, VLADIMIR KOSMA; RAMBERG, EDWARD GRANVILLE AND FLORY, L. E. Television in Science and Industry. 1958.

ENGINEERING

INSTITUTION OF MECHANICAL ENGINEERS AND THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. International Conference on Fatigue of Metals; Proceedings. n.d.

SIEBEL, ERICH AND LUDWIG, N. Prüf- und Messeinrichtungen. Ed. 2. 1958.

HOROLOGY

ZERELLA Y YCOAGA, MANUEL DE. Tratado General y Matemático de Reloxería. 1791.

INDUSTRIAL MANAGEMENT

BULLINGER, CLARENCE EDWARD. Engineering Economy. Ed. 3. 1958.

DAVIDSON, ROBERT L. Successful Process Plant Practices; Operation, Maintenance, and Safety. 1958.

NIEBEL, BENJAMIN W. Motion and Time Study. 1958.

MANUFACTURE

AMERICAN SOCIETY OF TOOL ENGINEERS. Tooling for Metal Powder Parts. 1938.

BILFINGER, ROBERT. Leitfaden für Galvaniseure. 1957.

UDIN, HARRY; FUNK, EDWARD R. AND WULFF, JOHN. Welding for Engineers. 1954.

MATHEMATICS

HADWIGER, H. Vorlesungen über Inhalt, Oberfläche und Isoperimetrie. 1957.

HOF, HANS. Powers, Roots, Reciprocals; from .0001–1.0000. Ed. 1. 1957.

HOF, HANS. Seven Place Full Natural Trigonometric Tables: Tangent 0° to 90°. Ed. 2. 1956.

HOLTOUG, FRANK LOUIS AND SKOV-JØRGENSEN, H. Teknisk Matematik til Brug ved Maskinog Elektroteknika. 1956.

REINFELD, NYLES V. AND VOGEL, WILLIAM R. Mathematical Programming. 1958.

SALZER, HERBERT ELLIS; RICHARDS, CHARLES H. AND ARSHAM, ISABELLE. Table for the Solution of Cubic Equations. 1958.

METALLURGY

PEARSON, WILLIAM BURTON. A Handbook of Lattice Spacings and Structures of Metals and Alloys. 1958.

MINERALOGY

RAMBERG, HANS. The Origin of Metamorphic and Metasomatic Rocks. 1958.

MINING ENGINEERING

ERCKER, LAZARUS. Treatise on Ores and Assaying. 1951.

PHOTOGRAPHY

Photographic Sensitivity. Vol. 1. 1956.

PHYSICS

BINDER, RAYMOND CHARLES. Advanced Fluid Mechanics. Vols. 1–2. 1958.

CAUCHOIS, YVETTE AND OTHERS. Action des Rayonnements de Grande Énergie sur les Solides. 1956.

EASTHOPE, C. E. Three Dimensional Dynamics. 1958.

EMMONS, HOWARD W. Fundamentals of Gas Dynamics. 1958.

GAMOW, GEORGE. Matter, Earth, and Sky. 1958.

HEAT TRANSFER AND FLUID MECHANICS INSTITUTE. 1958. Preprints of Papers. 1958.

PENNING, F. M. Electrical Discharges in Gases. 1957.

PLANCK, MAX KARL ERNST LUDWIG. Physikalische Abhandlung und Vorträge. Vol. 1. 1958.

V.2
195

SCIENTIFIC INSTRUMENTS

PEARSON, ERIC B. Technology of Instrumentation. 1958.
ZILL, HEINZ. Messen und Lehren im Maschinen- und Feingerätebau. 1956.

SUGAR

DREYSER, C. F. Wie Müssen zur Zuckerfabrikation Bestimmte Rundkelrüben Gebaut Werden? 1836.
DUBUC, GUILLAUME. Mémories sur l'Extraction et sur les Usages du Sucre Liquide des Pommes et des Poires, avec l'Analyse Comparee de Cette Substance et de la Mélasse du Commerce. 1809.
LINCKE, C. A. Der Anbau und die Behandlung der Runkelrueben Als Nahrungsmittel fuer Unsere Viehstaeme und als Material zur Zuckerfabrikation. 1836.

MEMBERSHIP

The following elections were approved on September 11, 1958 by the Membership Committee by authority given to it by the Board of Managers.

June 1–30, 1958

ANNUAL

Joseph Adamson
Joseph H. Anderer
Franklin H. Andrew
Isaac L. Auerbach
George E. Bartol, Jr.
Gordon M. Bitterlich
Herbert A. Boas, Jr.
Wilkerson Boone
J. H. Bowen, Jr.
Morris A. Bowie
Alexander S. Budd
Richard S. Bunsick
Harry Chiddenton
James Lee Cox
John B. Cramp
H. Cullumbine
C. Laurence Cushmore, Jr.
M. Willard Davies
Joseph G. Denny, III
Joseph Moreau Denshaw
John M. Dickinson
Isadore Director
Rocco A. DiTaranto
William L. Doyle
J. N. Eddowes
Allen H. Ett
S. J. Fitzgerald
A. R. Fox
William A. Fraser
Charles D. Fridy
John H. Frome, Jr.
Frank H. Fromm, Jr.
S. DeWitt Fulton
Joseph W. Gallagher
Herman K. Gessner
Daniel L. Goffredo
David F. Gould, II
A. J. Greenough
Theodore C. Hangey
Donald J. Hart
George V. Haslett
Benjamin Ingber
John H. W. Ingersoll
Elizabeth Kline Jordan
Luther J. Katinos
Charles J. Kern
John J. Larkin
J. E. Lewis
Alexander J. Lipman
Lionel Lorant
James J. Mallon
A. C. Martinsen
Charles E. Mather, II
E. S. Mendelson
Edward A. Pizzi
William F. Polfus
Edward G. Ramp
Leonard John Rusciani
Robert M. Sanders
Harleigh V. Saurman
John R. Slotterback
Alec B Smith
Harry J. Smith
Perrin G. Smith
Stewart W. Speers
Leonard Starobin
Bruno John Talvacchia
Samuel C. Tease
Gabriel A. Torok
Samuel C. Waters
Russell E. Weinkauf
Matthew B. Weinstein
F. Weston Whittier
D. C. Will, Jr.
F. S. Williams
William W. Wilson
William A. Wurster
Frank Yarosh
John I. Ykema

EDUCATOR

Donald C. Dittmer
Marion Fay
William D. Frazier
Charles C. Smith
Harry Stanley

Membership (continued)

NON-RESIDENT

C. F. Lovely

July 1–31, 1958

ANNUAL

Harold L. Beggs
Robert M. Briggs
Robert E. Dibble
Elio DiRusso
Stephen Domako
F. Robert Dougherty
Lydia D. Draeger
Charles J. Fitti
Andrew A. Foley
George David Franklin
Wallace W. Fullerton
John J. Garland
Michael J. Giordano
Victor S. Gittens
Kenneth F. Glace
Joseph M. Gladeck
George J. Goldsmith
I. W. Gower
William M. Greenhouse, Jr.
Henry E. Hockeimer
Wesley L. Hoffman
William A. Howard
William F. Hyland
Richard O. Jacob
Jack M. Kendree
Raymond E. Kierstead
George J. Kreier, Jr.
Jack H. Kubanoff
Morris Levenson
Herbert S. Levin
Nathaniel Lieb
Irving P. Magasiny
S. A. Mascari
A. E. May
Louis R. McCreight
Joseph V. McNulty
Emanuel O. Munzer
Herbert D. Nickerson
Mollie E. Orloff
Robert A. Phair
Robert L. Ravel
Kenneth Richter
H. B. Saldin
John B. Santosuosso
K. N. Satyendra
Herbert B. Schiele, Jr.
Paul B. Venuto
Walter J. Vreeland, Jr.
Francis H. Weng, Jr.
A. C. Yarnall

EDUCATOR

David B. Geselowitz
G. Barrett Glover

NON-RESIDENT

J. R. Rumble

August 1–31, 1958

ANNUAL

Donald W. Adams
Joseph H. Adams, Jr.
Alexander E. Aikens, Jr.
Silvio L. Apolito
Robert F. Bundy
Fredy E. Christian
William W. Collins
Alan S. Compton
Lindell A. Davidson
Samuel Delfiner
Charles J. Donahue
Samuel J. Ellis
Virginia S. Erickson
Egils H. Evalds
John J. Foulkrod, III
Gilbert M. Frederick
Joseph R. Gahagan
Albert M. Garber
Theodore M. Gloeckner
Ernest Goldberg
Richer M. Goodwin
Douglas W. Gould
Elmer H. Gould
A. Stuard Graham, Jr.
William E Graul
James A. Graves
Terence Richard Gregory
Gustave L. Grundmann
Raymond A. Haas
Carl F. Haeussler
Walter W. Hammond
S. Ryland Hanger
Randolph G. Helfrich
P. G. Holt
Ceora T. Hufnagel
Homer Jensen
Albert J. Kilberg
Harold A. Kline
Paul E. Koehler
Joseph R. Lynch
Donald P. Macrae
Frederick L. Maltby
Richard T. Nalle, Jr.
Mildred M Redding
Louis Remmers
Paul J. Riley
Edgar F. Rulon
Kingsley V. Schroeder
Walter S. Sharshon
Roger S. Simard
William P. Smith
James M. C. Sommerville
Walter H. Sponagle
Victor G. Szebehely
Charles E. Welsh
William B. Whiting
Hugh Winn
Gordon J. Wiest
D. Robert Yarnall, Jr.
William C. Zeek

EDUCATOR

Denys Osmund Akhurst
William S. Parkes
Sherwood Henry Spadafora
Yao-nan Yu

Membership (continued)

NON-RESIDENT

Edgar H. Nelson, Jr.

Leora Straka

NECROLOGY

Edward Bartow '13
C. Barton Brewster '42
G. Harry Ditter '56
W. F. Durand '38
Alban Eavenson '36
W. H. Eisenman '43
Samuel M. Freemen '36
Edith Bolling Jones '36
Henry Knepper '54
Arthur N. Klein '53
John Kremer '39
H. R. Martz '45
William W. Omin '58
Samuel Rosenfeld '45
Stephen C. Rowan '39
C. Louis Seidenburg '56
Benjamin Sork '57
John F. Straitz '44
Franklin E. Talmage '43
Laurence G. Waddle '57
Isabel Muller Wetherill '36
Nathan H. Wolf '42
Theodore V. Wood '45

DO YOU KNOW OF THE FRANKLIN INSTITUTE'S NEED FOR GIFTS?

Since 1824 The Franklin Institute, through wise and conservative management, has been partially self-supporting.

As an old institution, established in the public interest, The Franklin Institute has tried to earn most of its cost by diligent and productive work. Like many other institutions of a non-profit nature, The Institute has, in these years of an expanding economy and rising spiral of costs, found it increasingly difficult to perform the broad educational services of which the growing nation and territories are ever in need.

Despite an erroneous public impression gained, perhaps, by the excellence and wide range of services performed, that it is a wealthy, richly endowed organization, The Institute always needs funds.

There is a warm satisfaction in giving financial support to an organization that will use it constructively to help advance and improve our type of civilization.

Your gift or bequest, large or small, will be deeply appreciated. We shall be pleased to give any additional information regarding gifts or memorials.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

Please write to: The Secretary, The Franklin Institute, Benjamin Franklin Parkway at Twentieth Street, Philadelphia 3, Pennsylvania.

THE FRANKLIN INSTITUTE LABORATORIES
NICOL H. SMITH, DIRECTOR

USE OF HIGH-VISCOSITY LUBRICANTS IN HYDROSTATIC BEARINGS

BY

N. E. SINDLINGER [1]

Hydrodynamic lubrication requires the relative motion of bearing surfaces to generate the pressure necessary to separate the surfaces with a full fluid film. At starting or at low speeds, the surfaces are in metal-to-metal contact. Hydrostatic lubrication, on the other hand, does not require the relative motion of the bearing surfaces. The hydrostatic bearing builds up a load-carrying film by being externally pressurized with a pump. Thus, the hydrostatic bearing is as effective at starting, or at low speeds, as it is under normal operating conditions. When full lift is attained in hydrostatic bearings, starting friction is, for all practical purposes, reduced to zero. Since there is no metal-to-metal contact, even at starting, a hydrostatic bearing has theoretically infinite life.

In the past, oil, air, and water have been used successfully as the lubricating media in hydrostatic bearings. This does not mean that these are the only lubricants to be considered for hydrostatic lubrication. Any viscous medium which can be pumped may be used in a hydrostatic bearing.

It should be recognized, however, that hydrostatic bearings require power for their operation. This power is needed to circulate the lubricant at the proper pressure and in the required volume. Not all applications can justify the extra cost of pumping power and equipment. For this reason, there are many occasions where hydrodynamic and other types of bearings are, and will continue to be, the most economical.

In considering the selection of hydrostatic bearings, the power necessary for operation is a very important factor. The viscosity of the lubricant is the single most important variable influencing the power required. Additional variables are bearing configuration, recess size, and location. A high viscosity lubricant will require less flow than a low viscosity lubricant; therefore, it will require less pumping power to maintain a given film thickness.

Present practice is to utilize comparatively low viscosity oils with thin films separating the bearing surfaces. If the films become large, the flow rates greatly increase. It seems desirable to use lubricants of higher viscosity so that the film in the bearings may be made larger without incurring the penalty of excessive flow. There are, however, no easily adaptable commercial hydraulic units that may be conveniently used for this purpose. While it is true that there are pumps for handling high viscosity lubricants, they are generally expensive, and

[1] Development Engineer, Friction and Lubrication Branch.

ELECTRICAL CONTROL DIAGRAM

HYDRAULIC SCHEMATIC

Rheostatic bearing.

require a separate piston and cylinder for each bearing recess, if individual flow control is desired. There appears to be a real need for (1) selecting high viscosity lubricants and determining their performance, and (2) developing new equipment at a reasonable cost for handling them. An application of a high viscosity lubricant (grease) has been successfully made to a small rolling mill, and a simple flow control system was designed and built at The Franklin Institute for this purpose.

The illustration shows the circuit for delivering a controlled quantity of grease to each of four hydrostatic bearings used in the rolling mill. The grease was an EP grease grade O which would just pour at room temperature. By utilizing grease instead of a low viscosity oil, the flow and power requirements were reduced by a factor of ten.

In order to maintain an equal film thickness in each bearing, it is necessary to control the volume of flow of the lubricant to each bearing. Since grease is a non-Newtonian fluid, the apparent viscosity is a function of the shear rate. In utilizing orifices or capillary tubes with a small enough diameter to restrict the flow and introduce a stabilizing pressure drop, the pressures required to start the grease flowing become excessive. For this reason, conventional methods of controlling the flow of a non-Newtonian fluid cannot be conveniently utilized.

In the system shown in the illustration, the flow control was attained by the use of conventional hydraulic cylinders. One side of each cylinder was filled with grease by using a low pressure, positive displacement pump capable of handling this high-viscosity material. Into the opposite side of the cylinder, a controlled flow of oil was delivered by means of a hydraulic pump and flow control valve. Thus, by controlling the flow of oil to each cylinder with flow control valves, the rate of displacement of the pistons could be varied. In turn, the flow of the grease could be controlled.

The chief disadvantage of this system is that a recharging cycle is required and that continuous operation cannot be obtained. In the case of the rolling mill installation, however, this was not objectionable, since the recharging time was designed to coincide with coil changes.

The advantages of the rheostatic or grease lubricated hydrostatic bearing, as compared with hydrostatic bearings using low viscosity oils, are low flow and power requirements, and the ability to use large clearances. There is also a safety feature in that if the pump fails, there is a time lag before the grease squeezes out of the bearings, and the surfaces come into complete solid contact.

It appears that the use of grease or heavy oils is quite feasible in hydrostatic bearings. With sufficient development work, bearing systems of the type described can permit starting under full load with negligible friction and would have, theoretically, infinite life—all of this at substantially lower cost of operation than with present hydrostatic bearings.

THE FRANKLIN INSTITUTE MUSEUM
A. C. CARLTON, DIRECTOR

IT'S ABOUT TIME

BY

THOMAS COULSON[1]

Man has been measuring the hours and the days from time immemorial, and the makers of timekeepers have been all sorts and conditions of men—magicians, priests, astronomers, locksmiths, jewelers, and village carpenters until the work was taken over by precision instrument makers. The varying devices that have been employed for measuring time form a fascinating introduction to the work of the modern watchmaker. Anyone who wishes to do so may see the milestones in the long history of timekeeping in an exhibit recently added to the Museum under the sponsorship of the Hamilton Watch Company.

The oldest method of keeping time in America was adopted by the Aztecs who made a calendar. A replica in miniature of this famous calendar is shown alongside a model of the Babylonian Hemicyclium from which the division of the day into twenty-four hours emerged. Striding through the years we arrive at the clepsydra, or water clock, which is almost as old as sundials, but more useful as it measured time when the sun was not shining. In its simplest form this was an earthen vessel with a hole at the bottom. The vessel was filled with water up to a certain mark and the water trickled out of the hole. The passage of the water marked the passing of time. As time passed, a great deal of elaborate workmanship was lavished upon the clepsydra. Not only was it furnished with a pointer moving over a dial to mark the hours but mechanism was introduced so that the hour was struck on a gong, cocks crowed, lions roared, and small figures were made to move.

These elaborations were naturally uncommon and were possessions only of the very wealthy, but King Alfred of England was responsible for introducing a much simpler and inexpensive method of measuring time. He set apart six candles to be burned each day. These were twelve inches long, divided into twelve equal sections colored alternately black and white. Three sections burned in one hour and, as wax candles could be made with a reasonable amount of consistency, this was a tolerably accurate means of measuring time to meet the needs of the people of that day.

Eventually the mechanical clock operated by weights or a pendulum was developed. Obviously these were not portable and to provide a busy man with a convenient means of keeping time which he could carry with him, the first watch was made by Peter Henlein, an inspired

[1] Director of Museum Research.

locksmith of Nuremberg. He used a mainspring in place of weights or pendulum. Because of the similarity in size and shape, these early watches were called Nuremberg eggs. They were more remarkable for their beauty of decoration than for their reliability.

Examples of these different types of timekeepers are included in this exhibit, but the sponsors have not been content to present visitors with a backward glance into the past. They show how simple artistic taste is being employed without any sacrifice of accuracy in making the watches of today. Of especial interest is the new electrical watch the sponsor company has developed.

If it is not possible to peer into the future to forecast the trend, an effort has been made to project visitors into space, which seems to be the next significant advance required for time measurement. One of the consequences of the Einstein Special Theory of Relativity is that as we approach the speed of light bodies will increase in mass, lengths in the direction of motion will shrink, and time will slow down until at the speed of light it will stop. This dilation effect is judged by some space scientists to indicate that this may be a way in which people will travel to distant stars, a journey which might normally require five hundred years of normal life time. By using speeds approximating to that of light, time will slow down for the space travellers so that they will live longer than those who remain on Earth. There may be some solace in learning that whether the life span of the individual will actually increase is a matter of speculation and controversy.

The general exhibits relating to time measurement will be found in the Astronomy Section, Third Floor of the Museum. Visitors who find interest in these various methods of time keeping are recommended to visit the Physics Section, where there is an unique display of clock escapements.

BOOK REVIEWS

NUCLEAR STRUCTURE, by Leonard Eisenbud and Eugene P. Wigner. 128 pages, 6 × 9¼ in. Princeton, Princeton University Press, 1958. Price, $4.00.

This book on nuclear theory is not a textbook nor a monograph. Instead the authors' aim, as stated in the preface, is "only to acquaint the reader with points of view which we consider useful and interesting. . . . The emphasis is on the description and comparison of the assumptions of various theories, together with a discussion of only some of the consequences of these assumptions." It is, to the reviewer's knowledge, the first work which attempts to give a rapid survey of nuclear theory for the purpose of providing orientation and perspective. Being mainly nonmathematical it can be read previous to detailed study or course work in the field and can serve to motivate and guide such effort. It will also be useful to those who wish to be brought up-to-date on aspects which have developed since the publication of excellent texts several years ago.

The attempt to be intelligible to the novice in nuclear physics and at the same time to lead the reader to an understanding of current theories and outstanding problems presents the major difficulty of exposition. Aside, perhaps, from the three-page discussion of nuclear size and the table of nuclear reactions on page 31, the first five chapters will be read impatiently by anyone with previous knowledge of the standard introductory topics covered. The meat of the book is contained in the remaining six chapters.

In chapter six the concept of nuclear model is introduced and the current models are classified according to whether they envisage chaotic wave functions ("powder" models) or planetary ones (independent particle or shell models). Wigner's supermultiplet theory is discussed. Chapter seven, one of the finest, describes and compares the L-S and j-j coupling shell models, recounts attempts at a theoretical basis for the coupling rules, and presents several fundamental theoretical problems faced by the independent particle approach. The discussion is nicely balanced between stating accomplishments and unsolved problems. There is a refreshing emphasis on the research yet to be done.

In contrast chapter eight, consisting of only six pages on the α-particle model, the rotational collective model of A. Bohr, and a comparison of the j-j and collective models, is disappointingly limited. It is somewhat lacking in drawing attention to the richness of the subject, particularly the wealth of theoretical problems, as indicated for example in Villars' article in the 1957 Annual Review of Nuclear Science.

Chapter nine on close collisions discusses resonances, radioactivity, the cloudy crystal ball (optical) model, and the intermediate coupling or giant resonance model. Aside from a few equations in chapter ten, this is the only place in the book where the reader is confronted with a mathematical formalism. The important R-matrix theory of Eisenbud and Wigner, and the very recent (1955) giant resonance model of Wigner, are given a skillful treatment which will repay careful study. One might have hoped for a few additional topics in this chapter, such as the nuclear photoeffect and the question of the distributions of level spacings, and for a definition of the now widely used term "strength function."

The next chapter, on surface reactions, suffers from the same brevity as that on the collective models. Very quick calculations of the angular distribution in stripping and the probability of Coulomb excitation are given, but one is not made to sense the wide variety of direct processes.

The final two chapters contain clear explanations of the uses of studies of beta decay and electromagnetic transitions. A nice discussion is given of some of the problems posed for the theory of nuclear structure by the experimental results. The recent discovery of the lack of invariance of the beta interaction with respect to spatial reflection and charge conjugation has not been incorporated into the text, but this does not affect the bearing

of beta decay on nuclear structure to any great extent.

One of the most exciting features of the book is its list of references and historical evaluations. Many classic papers are listed and their significance pointed out. Of the more recent work, a choice selection is made. The excellence of this critical bibliography cannot be over-praised.

The material in this book will also appear as a section of a large "Handbook of Physics," edited by E. U. Condon, to be published by McGraw-Hill.

RICHARD L. BECKER
Oak Ridge National Laboratory

EFFECTS OF RADIATION ON MATERIALS, edited by J. J. Harwood, Henry H. Hausner, J. G. Morse and W. G. Rauch. 355 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1958. Price, $10.50.

During the last decade it has become popular to publish the proceedings of research conferences held on specialized topics. Sometimes these volumes are only of interest to the expert in serving as a convenient accumulation of research papers. The volume under discussion is different insofar that it contains *review* articles given at a colloquium which was planned to cover the most important aspects of radiation damage by nuclear particles. The eleven articles contained in it were written by leading research scientists and therefore the subject is presented without loss in depth. The papers deal with the effects of radiation on metals, semi-conductors and organic substances as well as the technical aspects that are of importance with reference to reactor techniques.

About two-thirds of the book are concerned with fundamental research problems in the field of solid state physics, while the remainder is nearly equally divided between effects on reactor materials as they are used in practice and organic substances. The first two articles which are also the longest deserve special mentioning. G. J. Dienes' paper gives a somewhat condensed but excellent review on current concepts of radiation effects. This allows the other authors to start immediately with their specific subjects such as radiation effects on metals, semiconductors, electronic and optical properties of inorganic dielectric materials, and the influence on corrosion. The second paper by J. C. Wilson is unique because it probably is the first publication which covers the details of experimental techniques for radiation studies. It should be of great value to both the experimentalist at the reactor and the scientist who wishes to have his materials irradiated. In the two articles on reactor components, radiation effects on cores, liquid coolants, control rods and shielding are discussed. The three papers on organic materials reflect very well indeed our present position in radiation chemistry. A specially compiled bibliography gives references of nearly 800 publications, ending at the beginning of 1956.

There can be no doubt that this conference was most appropriately timed: the research in the past has led to the realization of the problems and many basic experimental facts have been established. The book represents a most welcome summary of our present knowledge of radiation effects and should not lose its value for many years to come.

H. G. F. WILSDORF
The Franklin Institute Laboratories

THE ULTRA HIGH FREQUENCY PERFORMANCE OF RECEIVING TUBES, by W. E. Benham and I. A. Harris. 173 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $6.50.

The authors of this book have compiled an excellent summary of the effects of electron transit time on the performance of ordinary vacuum tubes at ultra-high frequencies. The material is presented in a very understandable manner and requires only a basic knowledge of the physical principles involved. Throughout the work, the practical significance of the results has been emphasized and the effects of approximations discussed. Numerous examples and graphical illustrations of the results have been included, which allow the reader to estimate the importance of each of the effects.

Essentially the same basic approach has been used to analyze each of the cases considered—calculation of currents and voltages from the fields existing in sections of the vacuum tubes. From these, expressions for the equivalent impedances by which the tube can be represented are obtained. A short discussion of the physical processes which form the basis of the theory is first presented.

The inadequacy of the low frequency approach is explained, and an equivalent circuit suitable for high frequencies is developed.

This is followed by a derivation of the equations which describe the currents in an interelectrode space, which are used later to treat the more complicated cases of triodes and tetrodes. The mathematics is limited to the basic electromagnetic laws, with emphasis on an understanding of the physical principles rather than an involved mathematical process. For this purpose, a simplified model of the electrode system is chosen, which, in spite of the restrictions placed on it, still retains sufficient similarity to actual systems to give accurate results.

The small-signal behavior of diodes and multi-element tubes is then derived. The multi-element tubes are treated as a combination of several diodes, coupled by means of the electron stream. The results are presented as a set of impedance functions describing the equivalent circuit. From these, the input impedance of several common circuit configurations is derived. A discussion of the effects of initial electron velocities and irregularities caused by grid structure is also included.

The remaining chapters are concerned with the effects of a retarding field and reflexion in diodes, large-signal analysis of diodes, and high frequency effects on the noise factor of vacuum tubes. The subject of velocity modulation is treated only to the extent that it is important in "ordinary" receiving tubes. Since adequate analysis of the klystron has been given elsewhere, it has not been included.

This book should be of great value to anyone who has only a limited knowledge of high-frequency vacuum tube theory because of the unified picture of the problem that is presented. The numerous references cited enable the reader to investigate any of the particular subjects in more detail than is allowed by the scope of the book.

W. C. REISENER, JR.
The Franklin Institute Laboratories

PRINCIPLES OF NOISE, by J. J. Freeman. 299 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $9.25.

Difficulties with noise became manifest shortly after the invention of the triode, when large factors of amplification made possible the observation of minute fluctuations in current and voltage previously undetectable. These random fluctuations today present a fundamental limit to the detection of signals and accuracy of measurements, and consequently have been the subject of considerable study over the past two decades.

There are relatively few general texts on noise available. Perhaps this is because the study of noise requires an extensive background in mathematics, probability theory, random processes, and circuit analysis, each of which constitutes an extensive field in itself. *Principles of Noise* by Dr. Freeman is intended as a self-contained text for graduate engineering students designed "to acquaint the student with enough of the principles, facts, and techniques used in noise analysis to take him to the level where he can read the literature with enough ease to use it as a professional tool." In spite of the difficulties involved, the book does perform its stated purpose.

The first three chapters of the book constitute a review of Fourier series and integrals, probability theory, and stationary random processes, equipping the reader with all of the tools needed for the noise analyses presented in the rest of the text. Here various physical sources of noise such as resistors, diodes, triodes, etc. (transistors omitted) are introduced together with the theory of their operation. Noise behavior of various networks is covered with the aid of equivalent noise generators; and the concept of noise factor is introduced and applied by way of illustration to cathode follower, grounded grid, and Wallman amplifying circuits. Two chapters are devoted to the very important subjects of minimum detectable signals and the influence of noise on the accuracy of measurement. Also included is a description of the influence of filtering and averaging, and the equivalence of these processes in improving the signal to noise ratio.

Because of its broad scope and the timely importance of its subject, *Principles of Noise* should be well received. The text will be valuable in the schools because of the difficulty of teaching a subject where most of the reference material is widely scattered through the literature. Dr. Freeman has included a number of references to important papers in the literature as well as a number of exercise

problems designed to illustrate and extend the material in the text.

The book is not elementary, and therefore is not recommended to those not having a good basic background in mathematics and engineering. For the graduate student and engineer it provides an understandable background of noise theory and analysis. Because of its broad coverage and good survey of Fourier series, probability theory, and random processes, it also serves as an excellent reference text.

D. L. BIRX
The Franklin Institute Laboratories

ZONE MELTING, by William G. Pfann. 236 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $7.50.

With the advent of semi-conductor technology and applications came the requirement for ultra-pure materials. The problem of large-scale production of these materials has been solved, for the most part, by the zone-refining process. While the periodical literature on zone-refining has grown to an appreciable size, there has been for some time a need for a general synthesis of the subject. This need has been successfully filled by W. G. Pfann's book, the first in the field. Mr. Pfann's treatment is quite vigorous and will satisfy both the scientist and engineer desiring a comprehensive knowledge of the subject. Those already working in this field will find the ideas on techniques useful in their own work.

The first three chapters deal exclusively with the theory of crystallization and its application to the zone-refining technique. Liberal use of charts and diagrams and the complete description of otherwise complicated equations brings the theory within the comprehension of the novice. Chapter 4 is devoted to the techniques of zone-refining and is quite complete in its description of presently used and suggested means of setting up the process. This chapter is in itself a valuable laboratory reference. Chapter 5 deals with the application of zone-refining to particular materials, including semi-conductiong elements and compounds, metals, and chemicals, both organic and inorganic. Some of the properties before and after zone-refining are described. The last section of this chapter presents general comments about the analytical methods used to detect impurities and their concentrations in ultra-pure materials. The remaining four chapters (6 through 9) describe the application of particular techniques to zone-refining and the application of zone-melting to requirements other than purification. These include many sound ideas for techniques of continuous zone-refining, zone-leveling, single crystal growth, perturbations of the concentration, and temperature-gradient zone melting. The reference list at the end of each chapter comprises a very good bibliography of most of the work that has been done in this field. The appendix is particularly valuable, presenting twenty zone-refining curves of relative solute concentration as a function of distance in zone-lengths for various numbers of zone passes in infinite and semi-infinite ingots.

Mr. Pfann's book, it is felt, will prove to be a valuable text for the student as well as a useful reference for the worker in the field of materials purification and preparation.

JOHN R. WIESE
The Franklin Institute Laboratories

THE POWDER METHOD IN X-RAY CRYSTALLOGRAPHY, by Leonid Azaroff and Martin J. Buerger. 342 pages, plates, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.75.

Recently the use of analytical X-ray diffraction techniques in research and industry has become widespread. This has created a need for trained X-ray diffraction technicians. Unfortunately, the number of available technicians has been inadequate, and often the budding X-ray diffractionist has had to acquire the knowledge and experience on a "do-it-yourself" basis. He luckily has had as guides several excellent texts on X-ray diffraction techniques. The powder method, however, is of such importance that it certainly deserves a separate treatment. The present volume is a welcome addition to the literature available for self training, and the authors are to be complimented for providing information on the powder method clearly and concisely.

The powder method, as the name implies, concerns the X-ray diffraction pattern produced by an aggregate of small crystallites. Two detection techniques are available: one

V.2
195

uses the diffractometer with the associated electronic equipment, while the other uses a camera with X-ray-sensitive photographic film. In this book the authors are concerned chiefly with the film method and in particular with a Debye-Scherrer type cylindrical camera. The principles discussed are, however, equally suited to the powder method in diffractometry.

Some of the chapters contain discussions of subjects which have been presented elsewhere. This could not be avoided, since in order to present a unified approach, some duplication was necessary. The first six chapters are more than an adequate summary of the basic principles. Elements of camera design, and the proper choice of equipment and radiation are among the topics discussed in these opening chapters.

One of the most important problems in the powder method is the determination of the crystal system from the interplanar spacings obtained from measurements of the diffraction lines on the powder photograph. This aspect is treated in detail in chapters 7, 8, 9 and 10.

In these the graphical and analytical indexing methods are clearly presented and a number of examples worked out. The use of the reciprocal lattice method receives the most attention. This is justly so since this method is suitable for most simple structures.

Chapters 11 and 12 are somewhat unique. The use of reduced cells is not too well known, and the treatment offered here is suitable for the novice as well as the experienced X-ray diffractionist. Much of this subject previously could only be found in the literature. Its inclusion in this volume offers the opportunity for greater application.

The Debye-Scherrer method is often used for the determination of precise lattice parameters. Certain errors are often encountered, and the elimination of these errors is necessary for obtaining high accuracy lattice parameters. Chapters 14 and 15 are devoted to the nature and the minimization of these errors. The presentation is inclusive and more understandable than the discussion found in most texts.

The remaining chapters discuss the identification of unknown materials by the powder method, and the appearance of powder photographs for various effects encountered in practice. Several useful tables are included in the appendix.

The addition of this book to a library on X-ray diffraction is recommended.

E. KLOKHOLM
The Franklin Institute Laboratories

AUTOMATIC CONTROL: PRINCIPLES AND PRACTICE, by Werner G. Holzbock. 258 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1958. Price, $7.50.

Generally, there is a broad technical division between the designers of high-performance servo-systems and those concerned with process control systems. Although they all deal with feedback controls, the method of expression of performance separates the two groups. On one hand, everything is described in differential equations, transform calculus, and complex plane plots, and on the other, behavior is expressed in more practical physical and algebraic terms. It is more difficult to cover adequately the complex field of automatic control systems with the latter method but the author has made this his goal.

The author is well qualified to prepare a book of this nature. In 1955, Reinhold published his previous book "Instruments for Measurement & Control." He has wide experience in the field, has published a number of papers in various technical journals, and has given many talks before ISA and ASME groups. Some of the material and its method of presentation form the background for this text.

The book is limited to the field of process control and instrumentation and is an excellent reference for the practicing engineer. The presentation of the subject material is intended to be within the grasp of the intelligent technician.

The author begins with a chapter on fundamentals in which basic concepts are defined. Terminology is according to ASME recommended standards and it is indeed unfortunate that it differs so widely from AIEE notation. The next two chapters extend the fundamentals to the description of the static and dynamic behavior of control system components. The treatment is in general terms: resistance, including thermal, fluid and electrical; capacitance, including thermal, volume, weight, and electrical; and time constants, including combinations of these. Through the medium of many well chosen illustrations, abstract terms are reduced to practical concepts.

In the next group of chapters, methods of testing, namely step response and frequency response, are described in detail, including the significance of the results to control system adjustments. Six basic types of controller action are covered and illustrated with actual components and typical responses. The author attempts a sketchy treatment of Nyquist diagrams, but recovers nicely with the application of less abstract Bode diagrams, in the analysis and synthesis of stable loops.

The following six chapters cover components, including mechanical and electrical devices, balancing and computing circuits, transducers, final control elements, and typical controllers. This section is a gold mine for even the most sophisticated servo-mechanisms engineer, with its descriptions of the action of a great number of commercial components for process control.

In closing, the author sums up with two chapters entirely devoted to complete control systems and their application in several industrial processes.

The text includes the statement "the purpose of an automatic controller is to reduce the error to an absolute minimum with the least possible delay." By this definition, the book itself qualifies as an excellent automatic controller by presenting its subject in the most direct manner.

CHARLES A. BELSTERLING
The Franklin Institute Laboratories

LEITFADEN ZUR BERECHNUNG VON SCHALLVORGÄNGEN, by H. Stenzel, revised by O. Brosze. 167 pages, diagrams, 6 × 9 in. Berlin, Springer-Verlag, 1958. Price, DM 31.50.

The book was first written in 1939 by Dr. H. Stenzel. The new, second edition by O. Brosze, has eliminated some mistakes and typographical errors and expanded the scope of the book by some added paragraphs.

The complete treatment of even simple sound fields poses considerable difficulties. This book wants to fill the need for a systematic development of the basic formulations which are then applied to simple examples.

The book starts with the treatment of the far field. In this case, when the distance from the sound source is large, considerable simplification can be obtained permitting an easy transition from the non-directional to the directional sound source. With this restriction, one can also calculate the sound field of membranes which do not oscillate with a constant amplitude.

In the second part of the book, the near field is treated. The field is presented by lines of constant sound pressure or frequently also by the lines of constant phase. The combination of two point sources and the field of a line source are being discussed.

In the third section the sound field of a simple and composite spherical sound source is calculated as well as the disturbance caused by a rigid sphere and a rigid infinite cylinder.

The theoretical derivations are limited to the extent necessary for the understanding of the subject with references to the pertinent literature for the one who wants to go deeper into the subject. The book has plenty of graphs and tables which permit the calculation of the sound field for a desired case without any major difficulties. The book is a real help to the sonic engineer and anyone who concerns himself with the subject matter.

KARL SITTEL
The Franklin Institute Laboratories

CERAMIC FABRICATION PROCESSES, edited by W. D. Kingery. 235 pages, illustrations, 8×10½ in. New York, Technology Press of Massachusetts Institute of Technology and John Wiley & Sons, Inc., 1958. Price $9.50

This book belongs in the series "Technology Press Books in Science and Engineering." It consists of 22 chapters by 19 contributors out of 22 scientists and engineers who took part in the 1956 Special Summer Program In Ceramics at MIT. It is stated in the preface that "Although presenting material by several different authors in symposium form loses some of the continuity that might otherwise be obtained, it has the great advantage that each section is authoritative and an integral unit. Consequently, the present book is not a review or a definitive text, but rather an attempt to evaluate the significance of current developments and indicate probable directions of future progress."

The contents are divided into six parts with from two to five papers in each. References are given at the end of each paper and an author index is provided as well as a short but satisfactory subject index.

After reading through all of these papers with their varied approaches and styles, one feels that a good balance has been achieved between presentation of the theory and of the practical fabrication methods now in use in industry as well as those newer fabrication methods which seem very strange to older ceramists.

Part I covering "Slip Casting" in 48 pages is nearly the longest in the book. To this reviewer, its content is the best known. It covers not only the theory and practice of preparing slips but also the complete preparation of plaster molds. In view of the very wide use of soluble silicates in slip casting, the lack of general reference to soluble silicates seems hard to explain except that in the research approach the soluble silicate systems are difficult to handle.

Part 2 covers the newer methods of pressure fabrication in some 26 pages and four papers and this is followed by Part 3,"Plastic Forming" with an additional four papers and about 33 pages. These are primarily concerned with the details of production using dry pressing, hydrostatic molding, and laboratory compaction in the first group and extrusion as well as the ordinary jiggering procedures in the second. Problems of non-clay systems are featured.

Part 4, "Drying and Firing," is covered by three papers in about 32 pages. These papers have to do with the theoretical reasons for warping and sintering and the possible application of this theory. Some simple mathematical relationships are developed.

Part 5, "Some Special Processes," is the longest section covering some 54 pages with four papers. The problems of hot pressing of ceramics are covered in considerable detail and related to the individual refractory oxides now in use. Again some simple mathematics is employed in an attempt to clarify the relationships.

The paper on cementitious bonding is a good review of cement as "a bonding material formed by reaction which is capable of imparting strength and elasticity to a body of aggregate in which it is present in minor proportion while detaining in part the properties of the aggregate."

A very short paper explains how nucleation works and how it is applied primarily, now, in the development of photo-sensitive opals.

The last paper in this section briefly reviews the application of metal-working processes to ceramics and cermets.

The last section, Part 6, "Ceramic Microstructures" is covered in about 30 pages by two papers, one directed to the electrical and magnetic properties and the other, the mechanical properties of ceramics. In general these are theoretical and represent developments which will become of greater importance as more is learned of the effect of microstructure.

The book is very well prepared and has only a few minor typographical errors. Anyone interested in ceramic properties is sure to find helpful comments or concepts well expressed by these authors who are truly outstanding in their field.

J. H. WILLS
Philadelphia Quartz Company

BOOK NOTES

SCIENCE AND EDUCATION AT THE CROSSROADS, by Joseph W. Still. 140 pages, 6 × 9 in. Washington, Public Affairs Press, 1958. Price, $3.75.

No interested person—whether scientist, teacher, student, politician or plain taxpayer—will read this book without realizing that the author has something original to add to the current science-education controversy. In contrast to most critics, Dr. Still goes beyond the "what's wrong" problem to propose solutions, both a short range one and a long-range one. His proposals may seem drastic to some, but will be applauded by others; for example, he proposes using a program of biological aid to underdeveloped countries as the most realistic road to peace.

In this book, Dr. Still presents the viewpoint of a working scientist on the needs of American education and foreign relations. He is well qualified to speak on both subjects; he does so with conviction and with a refreshing literary style that captivates the reader, even though he may not agree with the contents.

LOOKING AT THE STARS, by Michael W. Ovenden. 192 pages, diagrams, plates, 5 × 7¼ in. New York, Philosophical Library, 1958. Price, $4.75.

Written to introduce readers to the pleasures of astronomy, this is one of a series of books on leisure-time activities. The author begins with a comprehensive story of naked-eye astronomy, illustrated with star maps and diagrams showing the motions of the heavenly bodies. Proceeding to a discussion of telescopes, the author describes their construction and what can be learned about astronomy through their use. The recent discoveries made possible through radar, radio waves, rockets, etc., bring the work up-to-date. Twenty-eight excellent plates are used to advantage throughout this compact, but comprehensive introduction to astronomy.

ELEMENTARY STATISTICAL PHYSICS, by C. Kittel. 228 pages, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $8.00.

Aimed at beginning graduate students in physics, this book covers several areas of statistical mechanics, including stochastic processes and transport theory. The author develops the fundamentals of the subject using Gibbs' method of ensembles. The three parts of the text cover Fundamental Principles of Statistical Mechanics; Fluctuation, Noise, and Irreversible Thermodynamics; and Kinetic Methods and Transport Theory. Phase transitions and cooperative phenomena are not discussed. The problems and examples are applicable to nuclear physics, electrical engineering, solid state physics, metallurgy and chemistry. Prerequisites for the course include thermodynamics and modern atomic physics.

ADVENTURES IN THE WORLD OF SCIENCE, by Charles G. Abbot. 150 pages, 6 × 9 in. Washington, Public Affairs Press, 1958. Price, $3.50.

This is the autobiography of a man who, through 63 years of association with the Smithsonian Institution, has had unequalled opportunity to study America's scientific progress. The names of great scientists, explorers, Presidents and politicians are found on these pages of personal reminiscence. The book, an interesting combination of trivial sidelights and important highlights, is vivid testimony to the wide scope of Dr. Abbot's interests.

ENGLISH-RUSSIAN, RUSSIAN-ENGLISH ELECTRONICS DICTIONARY. 943 pages, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.00.

Prepared for the Department of the Army, this comprehensive electronics dictionary was compiled from modern sources available in the U.S.A., the United Kingdom, and the U.S.S.R. It translates approximately 22,000 Russian terms and abbreviations and 25,000 English ones. The dictionary was published primarily to aid the English-speaking researcher, translator, writer or engineer who uses Russian publications on electronics. In addition to the strictly electronic fields, the dictionary covers atomics, navigation and some physics terms. The book uses the ingenious edge index, which certainly cuts the cost of production, but is considerably more difficult to use than the usual thumb-index.

OSCILLOSCOPE TECHNIQUES, by Alfred Haas. 224 pages, diagrams, $5\frac{1}{2} \times 8\frac{1}{2}$ in. New York, Gernsback Library, Inc., 1958. Price: $2.90 (paper); $4.60 (hard cover).

The author, a French electronics engineer, has written a practical book for those who use the oscilloscope, either professionally or experimentally. Ten chapters cover a description of the instrument with its circuitry and accessories, and its uses in such fields as measurement of electrical magnitudes, amplifiers, tuners, etc. A final chapter is devoted to oscilloscope fault patterns. The best feature of the manual is the inclusion of several hundred reproductions of actual oscillograms with explanations of the patterns.

TECHNICAL EDITING, edited by B. H. Weil. 278 pages, $5 \times 7\frac{1}{2}$ in. New York, Reinhold Publishing Corp., 1958. Price, $5.75.

In September, 1957, the author served as chairman of a symposium on technical editing. The papers presented there are the basis for the present manual, directed at the men and women who devote full time to editing and publishing technical material. After an introduction setting forth the psychological, educational and professional aspects of technical editing, six chapters are devoted to editing of internal documents, four chapters deal with journal editing, two with books and manuals, three with the problems inherent in editing graphic aids, and three with miscellaneous editing problems.

Although the editor states that the manual is intended for the experienced technical editor, as well as for beginning editors and students of technical journalism, this reviewer questions whether the experienced editor will find much new material in the book. As a first book on technical editing, the manual should serve as a useful guide to those just entering the field.

CELLULOSICS, by Walter D. Paist. 270 pages, illustrations, $5 \times 7\frac{1}{4}$ in. New York, Reinhold Publishing Corp., 1958. Price, $5.75.

The major effort appears to be that of presenting as many examples as possible in a condensed book of the numerous applications of cellulosics. Several chapters have been devoted to showing how cellulosics are used in the forms of extrusions and castings, injection moldings and fibres. In three chapters, Protective Coatings, Adhesives, and Miscellaneous Applications, sample formulations have also been included. These are of value to those who have been intrigued with the properties of these resins as presented in the chapters on General Properties and Fabrication and who, not having any previous knowledge of these materials, wish to apply them.

The presentation of material is simple and direct and this will help the non-technical reader to a better understanding. There does not seem to be too much of value for the technical reader who has some associations with the cellulosics.

FLUOROCARBONS, by Merrit A. Rudner. 238 pages, illustrations, $5 \times 7\frac{1}{4}$ in. New York, Reinhold Publishing Corp., 1958. Price, $5.75.

Those who are interested in the practical applications of fluorocarbon polymers or who wish to learn more about this class of materials will find *Fluorocarbons* interesting. Information is presented on their general properties, chemistry, significance of processing and fabrication techniques and applications (mechanical, electrical and chemical). Each type of fluorocarbon is also identified by trade name and manufacturer. Of particular value is the comparison made between the properties and uses of various fluorocarbon resins.

This reviewer feels that the wear characteristics of Teflon, one of its most serious limitations, have not been sufficiently stressed. But for this, the value of fluorocarbons has been well presented.

INTRODUCTION TO NONLINEAR ANALYSIS, by W. J. Cunningham. 349 pages, diagrams, 6×9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $9.50.

This is the first book on nonlinear analysis written specifically as a textbook suitable for graduate courses. Most of the material, in fact, has been used in such courses at Yale University. Emphasis is on the use of mathematical techniques rather than on formal mathematics. Prerequisites for the course are electrical circuits, mechanical vibrations and linear differential equations.

The text itself covers numerical methods, graphical methods, equations with known

exact solutions, analysis of singular points, analytical methods, forced oscillating systems, systems described by differential-difference equations, linear differential equations with varying coefficients, and stability of nonlinear systems.

The problems for all chapters are collected in one section following the main text. The references are given an unusual treatment—instead of a list following each chapter, a special section on bibliography contains two parts: (1) a brief annotated bibliography with numbered references for each chapter; and (2) a list of 81 up-to-date references comprising those referred to in the annotated section.

SOLVING THE SCIENTIST SHORTAGE, by David C. Greenwood. 68 pages, 6 × 9 in. Washington, Public Affairs Press, 1958. Price, $2.00 (paper).

The author has prepared a well documented survey of the present state of scientific and engineering training programs, with emphasis on the programs being administered by government, industrial and professional agencies. In view of our current and urgent need for trained scientists and engineers, the book is a timely and sober contribution to the stream of criticism which followed the Russian launching of the first Sputnik.

The author divides his material into six chapters: (1) The Nature of the Problem; (2) The Educational Outlook; (3) Governmental Programs; (4) Private Programs (industry); (5) Professional Programs; and (6) Recommendations. Material was gathered in part through questionnaires. The final chapter on recommendations is an interesting collection of suggestions resulting from 1000 replies to a questionnaire sent to business executives, industrialists, educators, military commanders, civil servants, heads of laboratories, and officers of technical and scientific societies.

CURRENT TOPICS

Silicon Solar Cells.—Harvesting of the sun's limitless energy to serve man's electrical needs is the task assigned the unique silicon solar converters manufactured by Semiconductor Division of Hoffman Electronics Corporation, Evanston, Ill. Rays from the sun have long been used for heating and cooking, but direct conversion into electrical power is a present day achievement.

One of the most recent and dramatic applications for silicon solar cells involves the Navy's Vanguard satellite still circling the Earth. The 108 Hoffman-made cells located in six clusters placed symmetrically around the sphere's surface convert sunlight of outer space into electrical power which operates one of the satellite's radio transmitters.

The cells are also used in the Aerobee-High rocket as a "sensor" to activate a motion picture camera that photographs the Earth from extremely high altitudes. Outer space is black so the cells respond only to the relative lightness of the Earth. As the rocket spins, the camera, triggered by the cells, photographs the Earth.

The Vanguard satellite illustrates the military use of solar cells to provide power while the Aerobee-High shows their use as light detector devices which generate their own power.

The ability of Hoffman-made silicon solar cells to respond at a rate of 50,000 impulses per second makes them ideal for use in high speed air and ground computers. They may also be used as infra-red sensors and fire or flame detectors.

Solar converters are wafer-thin discs of extremely pure silicon fused with minute amounts of arsenic and boron. Arsenic and boron help create the electron groupings that provide positive and negative electrical fields.

After etching and plating, Hoffman technicians individually test each cell. The cells are then joined in series of various sizes and shapes to meet requirements of specific applications.

Light photons penetrating the silicon cause the flow of electrons and result in a voltage buildup within each silicon cell. This converted energy is then conducted to whatever outlet has been provided.

The world's first commercial solar cell panel, a 144-cell modular assembly made by Hoffman, is designed to supply electrical power in multiples of five watts. Remote areas without electrical power sources may rely on a system of these modular units for constant power.

Silicon solar cells have an estimated lifetime of 10,000 years, do not deteriorate, and are virtually unaffected by humidity and temperature variations. They also convert incandescent light into electricity.

Silicon solar converters made today have a top efficiency of about ten per cent. While not far enough developed at the present time to compete economically with high voltage electrical power, the converters are considered ideal for supplying low-power requirements in lieu of heavy, generated electricity.

Device for Testing Space-Age Electronic Equipment.—To test sensitive electronic gear for jet planes, missiles and rocket ships, engineers of the Radio Corporation of America have

developed a new device believed capable of producing the world's loudest controlled noise.

A. L. Witchey, engineer in charge of the project, said the new device—known as a Compressed Air Loudspeaker (CAL)—can generate noise of 160 decibels. He said this is more than 10,000 times as intense as the bedlam of the heaviest street traffic, and 20,000 times as great as that of the typical television set at maximum volume.

The Compressed Air Loudspeaker, now in the development stage at RCA's Industrial Electronic Products unit in Camden, New Jersey, is a plywood box measuring 5 by 5 by 6¾ ft. with dual horns resembling a pair of overturned pyramids projecting from the box. Attached to the tip of each pyramid is a metal pipe leading to the CAL's "plumbing system."

According to Mr. Witchey, acoustical research has shown that noise of high intensity affects more than the human ear. One of the most common effects, he said, is an intense vibration of the human body that may reach a point that becomes unbearably painful.

"Mechanical and electronic equipment," said Mr. Witchey, "reacts in much the same manner when subjected to a violent buffeting by high-intensity noise. Not only is this buffeting likely to cause actual shattering of delicate parts but tests have indicated that equipment often fails to perform properly in an atmosphere of extreme noise. An altimeter, for example, which has not been constructed to withstand the pounding of high-intensity sound, may give the jet pilot a false reading."

RCA, a leader in acoustics engineering for thirty years, first developed high-intensity noise systems using electrical power alone. These systems simulated "in-flight" noise levels up to 145 decibels—high enough for many test applications—but the demand for even higher noise levels continued. In their efforts to produce even greater sound with a relatively simple system, the RCA engineers turned to compressed air.

The CAL unit has attained sound pressure levels of 160 decibels, against the 145-decibel level of standard noise systems. However, the increase of 15 decibels doesn't tell the full story; each time a rise of three decibels is recorded, the power of the system is doubled.

While the RCA modulated compressed air noise system is considered of great value for testing equipment, its developers also point to its possibilities as a loudspeaker. The dual horn unit is expected to prove useful in areas of extremely high noise levels—such as jet airfields, the flight deck of a jet-age aircraft carrier, in certain types of industrial plants, and in civil defense work. If opened up to its maximum capabilities, engineers believe that CAL could project the human voice a distance of ten miles or more.

Nitrile Silicone Rubber.—A new kind of rubber discovered by General Electric research combines oil resistance with the ability to maintain strength and usefulness at temperatures ranging from a sub-arctic 100 degrees below zero (F.) to the 500-plus degrees required for modern jet aircraft.

Nitrile silicone rubber was described as "a major advance in chemistry that will offer industry—particularly the automotive and aircraft industries—a unique combination of important properties never before available in a single material."

The announcement was made by Dr. Guy Suits, General Electric vice president and director of research, and by Dr. Charles E. Reed, general manager of the company's Silicone Products Department.

At a demonstration, representatives of the press saw samples of nitrile silicone rubber remain calmly impervious when immersed in hot jet fuels that caused ordinary rubber materials to writhe, curl, and swell into ugly shapes much larger than their original size.

Dr. Reed stressed that the new nitrile silicone rubber can be processed in ordinary rubber fabricating equipment. He said it will be basically a less expensive material than other rubbers which offer oil- and heat-resistance only at the sacrifice of other important properties.

"We believe that the uses for nitrile silicone rubber will be almost unlimited," Dr. Reed said. "The Air Force and the aircraft industry have long sought a material of this type. Many O-rings, seals, and gaskets in aircraft engines and hydraulic systems are in constant contact with fluids. As aircraft speeds continue to increase, many of the materials now used for these parts will not be adequate to meet the increased temperature requirements."

One of the most important automotive applications will be in automatic transmission seals where nitrile silicone rubber will "eliminate the need for compromise" between marginal oil resistance with some materials and marginal heat resistance with others. Differential seals, crank-shaft seals, rubber brake parts, and gaskets also promise to be automotive applications, Dr. Reed said.

Other industries cited as potential users of the new material include electrical apparatus, railroads, oil-well operations, and many other kinds of lubricated industrial equipment that use gaskets and seals.

Dr. Reed indicated that nitrile silicone rubber would be available as a product before the end of 1958. The introductory price is expected to be in the vicinity of fifteen dollars a pound.

Basic chemical research in "polar" silicones which resulted in the introduction of nitrogen in the form of nitrile groups into silicone polymers was performed by Drs. Maurice Prober and Glenn D. Cooper at the General Electric Research Laboratory.

"Polar groups," Dr. Prober explained, "strongly attract each other. We felt that these polar groups would radically alter the swelling properties of silicone rubbers in fluids such as gasoline. Many types of nitrile silicone rubbers have been prepared in the laboratory, covering a broad spectrum of properties."

The development of nitrile silicone rubber into a process suitable for commercial production was accomplished by Dr. Ben A. Bluestein and a group of his associates at the General Electric Silicone Products Department, by synthesizing the nitrile-containing silicone. He told how this pure nitrile intermediate is converted to a fluid by hydrolysis and then to a very high molecular weight polymer. The long chain polymer is converted to a rubber by conventional techniques.

"The use of the correct type and amount of filler and vulcanizing agent results in a material of optimum consistency for subsequent molding and extruding operations. The result is a rubber of outstanding properties," he said.

New Lubricant.—A series of radiation-resistant fluids for possible use as lubricants, reactor coolants, or hy-

draulic fluids over a wide temperature range has been developed for the Air Force. Considered a major advance, polyphenyl ethers, composed of benzene and oxygen, remain liquid from 20 to 800° F. and retain lubrication characteristics throughout these temperature ranges. Higher temperature exposure is possible for brief periods.

Developed for the Air Research and Development Command, the polyphenyl ethers are two to five times more stable under nuclear radiation than existing organic lubricants. The new fluids were research projects completed by the Monsanto Chemical Company and the Shell Development Company for ARDC's Wright Air Development Center (WADC), Dayton, Ohio.

In thermal stability tests at WADC's Materials Laboratory, the polyphenyl ethers showed no decomposition below 800° F. The fluids are effective under severe oxidizing conditions at temperatures as high as 500° F. In high temperature coking tests up to 1000° F., they showed relatively minor coke formation.

Work is now underway to test the polyphenyl ethers in full-scale equipment under intense nuclear radiation. The tests will determine the fluids' effectiveness under combined heat and nuclear radiation environments.

Rand McNally Offers Map of the Moon.—Rand McNally & Co., which has pioneered in mapping the earth's surface, has announced publication of a map of the moon. Based on photographs to assure complete accuracy, the map gives a comprehensive picture of the moon's surface as seen from the Earth, showing and naming all important craters, mountains, oceans and valleys. It is the largest map of the moon to be produced in the United States, and one of the first.

When the first rocket is fired at the moon and hits the target, the point of impact can be readily traced on the new map. U.S. strategists claim a 70 per cent margin of success in the first attempt to hit the moon, and it is expected that a reflective substance will be released on the moon's surface so that the spot can be seen from the Earth.

In order to show greater detail which would be lost in a photographic reproduction of the full moon, the new Rand McNally map shows the two quarter views comprising the 59 per cent of the moon's surface visible from the Earth. Highlighting shadows on the quarter views make the surface clearly discernible. (The moon requires exactly the same time to circle the Earth as it does to make one full turn on its axis; thus, it always turns the same face toward the Earth and the reverse side is not visible. Slight wobbles in the moon's motion account for the visibility of 59 per cent of the surface from the Earth.) In addition, side panels on the map provide detailed, blown-up photographs of some of the major features of the moon. The new map also contains full-color drawings of the Jupiter and Vanguard rockets and the first American satellites.

On the reverse side is a map of the planets, showing their size and distance relationships, as well as information and diagrams illustrating the diameters of the planets and the sun, eclipses, tides, and seasons, the earth and its atmosphere, and Halley's Comet.

Rand McNally's map of the moon is based on photographs taken by the Lick Observatory, at the University of California, Berkeley. Enlargements of physical features are from photographs taken by the Mount Wilson and Palomar observatories.

The new map is 46½ by 26 inches in size, and folds to 6½ by 10½ inches. It is available in book, stationery and department stores, and sells for $1.

Improved Jet Landing Barrier Developed.—An improved emergency runway barrier system for stopping jet aircraft rolling at high speeds after a landing has been developed for the Air Research and Development Command (ARDC). The barrier can safely halt a jet fighter without braking power traveling at nearly 200 mph., a medium jet bomber at 170 mph., and heavier bombers at lower speeds.

Under contract to ARDC's Wright Air Development Center (WADC), the barrier was developed by the All-American Engineering Company, Wilmington, Delaware.

Carriage-like friction brakes operating along 200-ft. tracks flanking the runway connect a steel cable and a nylon net stretched across the runway. As the aircraft rolls into the webbing, its nose wheel is dragged along, pulling the cable up to catch the main gear.

The cable then pulls the carriages along the track as the braking action occurs. Through use of pulleys, the cable is arranged to move 1000 ft. while the braking carriages move 200 ft.

As the carriages move along the tracks, energy is dissipated by brake shoes riding between 14 stainless steel ribbons on each track. Each ribbon runs the length of the track and measures 10 in. wide.

New High-Temperature Alloy.—A new high temperature alloy has been developed by the General Electric Company in the battle to smash the thermal barrier. Trade-marked by the General Electric Company René 41, the new nickel base, titanium-aluminum hardened alloy will make possible higher jet engine and airframe Mach numbers at lower specific weight because of its high temperature capability.

Developed jointly by the General Electric Company's Jet Engine and Flight Propulsion Laboratory Departments at Evendale, Ohio, the new metal will be marketed by General Electric's Metallurgical Products Department with headquarters in Detroit, Mich.

Although René 41 high temperature alloy was designed for use in severely stressed jet engine components, its high temperature strength makes it ideal for many applications where high temperature resistance and performance reliability are required.

Despite its fairly large content of titanium and aluminum, René 41 can be formed, welded and machined with relative ease in comparison with other materials designed to operate in this temperature range.

Capable of use in the 1800-deg. F. range, which is considerably higher than currently used sheet metal of comparable rupture strength, the new alloy also can be used effectively through the entire range from room temperatures to 1800 deg. F. by appropriate heat treatment.

Welding offers no major problems to the new alloy. It can be inert-arc welded with or without filler material, as well as offering strong, ductile spot welds. Welds closely approach the strength of the base metal.

Because it is one of the strongest materials that can be successfully formed and welded, René 41 sheet is well suited for afterburner parts, turbine casings, nozzle diaphragm partitions and combustion liners. The mechanical properties of the metal are expected to make it an excellent high temperature bolting and fastener material and desirable for high Mach airframe components.

René 41 offers a 40 to 60 per cent improvement in sheet tensile strength over existing commercial types at 1000–1400 deg. F. yet it can be produced with relative ease in sizes required to manufacture large sheet metal components. Strength tests of the new metal with optimum heat treatment show that the 0.2 per cent offset yield strength at 1200 deg. F. is 125,000 psi. The 100-hr. stress rupture strength at 1600 deg. F. is 25,000 psi.

New 2000-Watt Ultrasonic Welder. —The welding industry has a valuable new piece of equipment in a 2000-watt welder, part of the SONOWELD line which utilizes ultrasonics to join similar and dissimilar metals without fusion, and with very little external deformation.

Aeroprojects Incorporated, of West Chester, Pennsylvania, who originated and developed this new joining method, has announced the marketing of a 2000-watt Model No. W-2000-SR-57-10 SONOWELD production type welder through its subsidiary, Sonobond Corporation.

A semi-portable unit, the W-2000-SR-57-10 SONOWELD has a welding frame of box-beam construction, and a hydraulic force system. Its transducer-coupling assembly consists of a nickel transducer, a coupling wedge and a drive-reed which have been especially designed for this unit. The heavy duty generator, designed for continuous duty is contained in two separate cabinets with ample spacing for components to allow adequate ventilation and insure high duty cycle operation, at the same time eliminating the danger of overheating. The welder is instrumented with an electronic cycle timer, and a Bourdon-tube force gage.

A spot-type welder, SONOWELD can be operated on an ordinary 230 volt circuit, and consumes a maximum of 3.5 KVA of power even when joining 0.050-in. 1100 aluminum sheet. Ultrasonic welding requires a great deal less electrical power than resistance welding, often as low as five per cent of that required by ordinary resistance welding. Since no electrical busbars or special transformers are required for the installation, this unit is mounted on casters so that it can be easily moved to and operated in various plant locations.

SONOWELD is so simple to operate that there is practically no training involved. The operator simply places the work to be welded between two welding members or sonotrodes, and activates a foot switch which initiates the entire weld cycle automatically.

Ultrasonic welding is a solid state joining process in which the materials to be joined are subjected to high frequency alternating vibrations, which are generated by the transducer in the welding head system and transmitted through the coupling members to the work being done. The resulting joints are accomplished without fusion, and, in many materials, exceed the strength of similar joints made by standard resistance welding methods.

An outstanding feature of SONOWELD lies in its ability to join dissimilar metals and alloys, and extremely thin gauges of materials which defy conventional welding methods. Materials that have been bonded together, either to themselves or to other metals include aluminum, copper, nickel, stainless steels, molybdenum, tantalum and niobium. Thicknesses to 0.050-in. of 1100-H aluminum and 0.025-in. of type 316 stainless steel have been bonded together in this size welder.